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Modeling Automation in Task Space

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Abstract

This paper introduces the task space model to analyze the impact of technological change on labor markets. It defines an n-dimensional task space where each task is a vector of attributes required for execution, as well as two dynamic subsets: the viable subset (tasks with profitable outcomes) and the automatable subset (tasks more profitable when automated). The framework provides a microfoundation for Acemoglu and Restrepo's displacement and reinstatement effects by tracking the dynamics of these subsets' boundaries based on individual task profits and relative factor productivities. This approach allows for analysis of how task similarities influence automation impact and aims to enable quantitative projections of occupation-specific susceptibility to automation.

1 Introduction

Automation and its consequences for labor markets have become central concerns in recent years. Advances in computer hardware, software, robotics, and artificial intelligence (AI) are enabling machines to perform an expanding range of tasks previously done by humans. Historically, automation technologies have primarily targeted tasks characterized by routine, codifiable procedures, whether manual or cognitive [7]. More recently, a wide swath of non-routine cognitive tasks have been rapidly automated by AI models to varying degrees of success, raising questions and anxieties about the future of work and how the benefits of new technologies are allocated between capital and labor.

This paper introduces the **task space model** as a method of quantifying the impact of technological change on labor markets. Our model intends to serve as a micro-level foundation for Acemoglu and Restrepo's [6] task-based framework, which analyzes economic production with tasks—the individual components of our occupations—as the fundamental unit of production. We define the task space in n dimensions, where each task is characterized by a vector with attributes mapping its execution requirements. Within this space, we define two key time-varying subsets: the viable subset, comprising tasks that result in non-negative profits, and the automatable subset, containing tasks technically feasible for automation.

The task space concepts provide a more granular view of the core ideas in the Acemoglu and Restrepo (hereafter, A&R) framework. The emergence and movement of tasks between the viable and automatable subsets within our model directly corresponds to A&R's concepts. When automation expands into regions previously performed by humans, we observe the displacement effect, where capital takes over tasks previously done by labor. Conversely, when new tasks are generated outside the automatable subset, we see the reinstatement effect where new task vectors populate the space. Unlike A&R's assumption that new tasks are initially performed by labor, our model determines factor allocation based on relative productivities and costs, allowing new tasks to be immediately automated when instantiated.

The model provides a structure to which we can map occupational and technological data, intended to enable projections of *which* tasks become susceptible to automation as technology evolves. Our approach offers two advantages over existing frameworks. First, it enables us to analyze and predict how the boundaries between human- and machine-performed tasks shift over time due to changes in technological capabilities and economic conditions. Second, it captures how similar tasks (those located close to each other in the space) may be similarly impacted by automation. By connecting specific task characteristics to economic outcomes, the model can help policymakers, businesses, and workers anticipate and adapt to technological change.

While the mathematical definitions and framework of the task space model are introduced in Section 3, formal results and estimations are omitted. This paper serves as a simple initialization of the conceptual architecture necessary for future theoretical and empirical development.

The remainder of the paper is organized as follows. Section 2 situates this work in the context of the existing literature on economic models of automation. Section 3 introduces the task space model by defining the foundational constructions and dynamics. Section 4 provides economic functional forms and interpretations for task space components. Section 5 applies these functional forms to illustrate how the canonical Acemoglu and Restrepo model can be represented in task space. Section 6 provides a stylized 2-dimensional visualization of the model. Section 7 concludes with a roadmap of theoretical development and empirical testing.

2 Literature Review

The task space model is directly inspired by the foundational task-based framework of A&R [6], which studies technological impact in terms of how tasks are allocated between labor and capital rather than through the undifferentiated factor augmentation of earlier literature [3]. A&R conceptualize production as occurring along a continuum of tasks, indexed on the unit interval $[N-1, N]$, with a threshold I determining which tasks are performed by labor versus capital. Their framework distinguishes between automation technologies, which enable capital to perform tasks previously completed by labor, and technologies that create new tasks where labor holds comparative advantage. This distinction induces two opposing effects: the displacement effect, in which capital takes over established tasks, and the reinstatement effect, where new tasks are created for which labor is assumed to initially hold a comparative advantage. A&R further derive how these task allocation dynamics directly affect the labor share of income through changes in the task content of production, providing a natural interpretation for observed labor income share declines.

Our model is further inspired by earlier work investigating the relationships between technology, tasks, and skills. Zeira [13] provided an early endogenous automation framework, showing how technology adoption, specifically the replacement of workers by machines, is more likely in high-productivity economies, thus amplifying cross-country differences. Acemoglu [1] extends this idea through the theory of directed technical change, which captured how the relative abundance of factors and their prices incentivize specific innovation trajectories, motivating our model's definition of economic thresholds in driving automation decisions. These works formalize how economic incentives shape not just whether automation occurs, but which specific tasks become targets for automation, a relationship explored in Sections 4 and 5 through our simple functional forms.

Autor, Levy and Murnane (ALM) [7] established the distinction between routine, non-routine, manual, and cognitive tasks that has become central to the empirical analysis of technological change and labor markets. They demonstrated that computerization primarily substitutes for routine tasks while complementing non-routine cognitive tasks, leading to a polarization in labor market outcomes. These dimensions provide natural axes for visualizing the task space, as we demonstrate in Section 6. While ALM's binary categorization effectively captures broad technological trends, the multi-attribute representation in our task space model aims to allow for more nuanced analysis of task similarity, partial automation, and the evolving boundaries between automated and non-automated regions. This approach enables the identification of "neighboring" tasks that may face similar automation risks due to their proximity in attribute space but are not captured when mapped to the traditional routine/non-routine or manual/cognitive categories.

Recent papers have refined and applied the task-based model. Restrepo [11] surveys theoretical and empirical developments, emphasizing how productivity gains and displacement interact. Acemoglu [2] applies a task-based lens to AI, disentangling which task types are most affected. Acemoglu and Loebbing [5] focus on polarization in labor markets, using a continuum of task complexity to explore "interior automation" and wage inequality. Acemoglu, Kong & Restrepo [4] further categorize technology types and their substitution patterns. Our formulation offers a complementary perspective to these models well suited for analyzing the role of task similarity and the evolution of the boundaries between automated and non-automated tasks in a multi-attribute space.

Multi-dimensional characteristic spaces have precedents in economic theory. Lancaster [9] pioneered the use of characteristic spaces to model consumer goods as bundles of attributes, enabling analysis of product differentiation and consumer choice based on preference for specific characteristics rather than goods themselves. Rosen [12] extended this approach to develop hedonic price theory, modeling goods as vectors of objectively measured characteristics that determine their market valuation. More recently, Lindenlaub [10] applies a multi-dimensional framework specifically to labor markets, representing both workers and jobs as points in a continuous attribute space to analyze sorting patterns based on comparative advantage.

3 Conceptual Framework

This section develops the mathematical framework of the task space model. We define the task space $\mathcal{T}$ and two component subsets, the viable subset $E(t)$ and the automatable subset $A(t)$, each defined by potential and threshold functions. The framework is designed to be general, allowing for various interpretations such as the one posed in Section 4.

3.1 Task Space Representation

Definition 1 (Task Space)**.** Let the task space be $\mathcal{T} = \mathbb{R}^n_{\geq 0}$. Each task is represented by a vector $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathcal{T}$, whose components x_i represent n distinct, continuously variable attributes characterizing the task, capturing fundamental characteristics required for task execution.

Let $M : \mathcal{T} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be the *task magnitude function.* $M(\mathbf{x}, t)$ quantifies the degree to which a task $\mathbf{x}$ is performed at time t. A task $\mathbf{x}$ at time t is *instantiated* when its magnitude flips from $M(\mathbf{x}, t - \epsilon) = 0$ to $M(\mathbf{x}, t) > 0$.

Proximity between task vectors in $\mathcal{T}$ corresponds to similarity in terms of these attributes. The dimensionality n reflects the chosen level of abstraction for characterizing tasks. The origin $\mathbf{0} = (0, \ldots, 0) \in \mathcal{T}$ represents a baseline reference task.

Definition 2 (Viable Subset)**.** The *viable subset*, denoted by $E(t)$, is the set of tasks within $\mathcal{T}$ that are viable at time t. The inclusion condition for $E(t)$ is determined by the inequality of two scalar functions Φ and Φ^{min} defined on $\mathcal{T}$.

Let $\Phi : \mathcal{T} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be the *viability potential function.* $\Phi(\mathbf{x}, t)$ quantifies the potential for a task $\mathbf{x}$ to exist at time t.

Let $\Phi^{min} : \mathcal{T} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be the *viability threshold function.* $\Phi^{min}(\mathbf{x}, t)$ quantifies the minimum potential required for task $\mathbf{x}$ to exist at time t.

The viable subset is then defined as:

$$E(t) = \{\mathbf{x} \in \mathcal{T} \mid \Phi(\mathbf{x}, t) \geq \Phi^{min}(\mathbf{x}, t)\}.$$

Definition 3 (Automatable Subset)**.** The *automatable subset* at time t, denoted by $A(t)$, is the set of tasks within $\mathcal{T}$ that are automated at time t. The inclusion condition for $A(t)$ is determined by the inequality of two scalar functions Ψ and Ψ^{min} defined on $\mathcal{T}$.

Let $\Psi : \mathcal{T} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be the *automation potential function.* $\Psi(\mathbf{x}, t)$ quantifies the potential for a task $\mathbf{x}$ to be automated at time t.

Let $\Psi^{min} : \mathcal{T} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be the *automation threshold function.* $\Psi^{min}(\mathbf{x}, t)$ quantifies the minimum potential required for task $\mathbf{x}$ to be automated at time t.

The automatable subset is then defined as:

$$A(t) = \{\mathbf{x} \in \mathcal{T} \mid \Psi(\mathbf{x}, t) \geq \Psi^{min}(\mathbf{x}, t)\}.$$

Basic Assumptions:

1. **Continuity of Functions in Space:** The functions $\Phi(\mathbf{x}, t)$, $\Phi^{min}(\mathbf{x}, t)$, $\Psi(\mathbf{x}, t)$, and $\Psi^{min}(\mathbf{x}, t)$ are continuous in $\mathbf{x} \in \mathcal{T}$ for each fixed t.

2. **Compactness of Subsets:** The viable subset $E(t)$ and the automatable subset $A(t)$ are compact subsets of $\mathcal{T}$, meaning they are closed and bounded.

   - **Closedness:** By Assumption 1, the continuity of the subsets' component functions implies the pre-images of closed sets are closed. Thus, $E(t)$ and $A(t)$ are closed subsets of $\mathcal{T}$.
   - **Boundedness:** $E(t)$ and $A(t)$ are assumed to be bounded subsets of $\mathcal{T}$ for all t, meaning there exists a constant $K > 0$ such that $\|\mathbf{x}\| \leq K$ for all $\mathbf{x} \in E(t) \cup A(t)$.

3. **Regularity of Functions in Time:** $\Phi(\mathbf{x}, t)$, $\Phi^{min}(\mathbf{x}, t)$, $\Psi(\mathbf{x}, t)$, and $\Psi^{min}(\mathbf{x}, t)$ are differentiable with respect to time t, allowing for dynamic analysis.

Recall from Definition 1 that $M(\mathbf{x}, t)$ measures the extent to which each task is performed. We define the non-automated and automated task shares as follows:

- The non-automated task share is $\Gamma(t) = \frac{\int_{E(t)\setminus A(t)} M(\mathbf{x},t)\, d\mathbf{x}}{\int_{E(t)} M(\mathbf{x},t)\, d\mathbf{x}}$, and
- The automated task share is $1 - \Gamma(t) = \frac{\int_{E(t)\cap A(t)} M(\mathbf{x},t)\, d\mathbf{x}}{\int_{E(t)} M(\mathbf{x},t)\, d\mathbf{x}}$,

By definition, $\Gamma \in [0, 1]$.

3.2 Dynamics

Let $e(\mathbf{x}; t) = \Phi(\mathbf{x}, t) - \Phi^{min}(\mathbf{x}, t)$ be the *viability difference function.* A task $\mathbf{x}$ is viable at time t if $e(\mathbf{x}; t) \geq 0$.

Let $a(\mathbf{x}; t) = \Psi(\mathbf{x}, t) - \Psi^{min}(\mathbf{x}, t)$ be the *automation difference function.* A task $\mathbf{x}$ is automatable at time t if $a(\mathbf{x}; t) \geq 0$.

We define $e(\mathbf{x}; t) = 0$ as the *viability boundary* and $a(\mathbf{x}; t) = 0$ as the *automation boundary.* Both $e(\mathbf{x}; t)$ and $a(\mathbf{x}; t)$ are continuous in $\mathbf{x}$ due to the continuity of their component functions.

The evolution of the task structure depends on changes to task viability, driven by $e(\mathbf{x}; t)$, and to automation status, driven by $a(\mathbf{x}; t)$. For any difference function $d \in \{e, a\}$ and its corresponding subset $S(t) \in \{E(t), A(t)\}$, the boundary dynamics are governed by the time derivative:

$$\frac{\partial d(\mathbf{x}; t)}{\partial t} = \frac{\partial F(\mathbf{x}, t)}{\partial t} - \frac{\partial F^{min}(\mathbf{x}, t)}{\partial t},$$

where $F \in \{\Phi, \Psi\}$ and $F^{min} \in \{\Phi^{min}, \Psi^{min}\}$ are the component potential and threshold functions of $S(t)$. Note that even if each component function of $S(t)$ is individually non-decreasing in t, their difference $d(\mathbf{x}; t)$ is not necessarily increasing in t.

The sign of this time derivative at boundary points where $d(\mathbf{x};t) = 0$ dictates the local dynamics:

- **Expansion:** If $\frac{\partial d(\mathbf{x};t)}{\partial t} > 0$ at a boundary point $\mathbf{x}$, the subset $S(t)$ expands locally at $\mathbf{x}$. The task $\mathbf{x}$ transitions into the subset at time t.
- **Contraction:** If $\frac{\partial d(\mathbf{x};t)}{\partial t} < 0$ at a boundary point $\mathbf{x}$, the subset $S(t)$ contracts locally at $\mathbf{x}$. The task $\mathbf{x}$ transitions out of the subset at time t.
- **Stasis:** If $\frac{\partial d(\mathbf{x};t)}{\partial t} = 0$ at a boundary point $\mathbf{x}$, the boundary is static at $\mathbf{x}$. The status of the task does not change at that time t.

Beyond simple boundary movements, both the viable subset $E(t)$ and automatable subset $A(t)$ may undergo more complex topological changes over time:

- **Merging:** Two previously separate components, $C_i(t)$ and $C_j(t)$, can merge if the condition $d(\mathbf{x};t) \geq 0$ becomes satisfied for tasks $\mathbf{x}$ along a continuous path connecting them.
- **Separation:** A single connected component $C(t)$ can separate into distinct components if the condition becomes unsatisfied ($d(\mathbf{x};t) < 0$) for tasks $\mathbf{x}$ within areas that previously linked parts of the component.

The stability of regions within each subset and their connections depends on the relative growth of the potential and threshold functions across the task space.

4 Economic Interpretation

We now provide economic interpretations for the functions and subsets introduced in Section 3 via simple functional forms for the viability and automation functions, followed by an economic interpretation of the subset dynamics.

4.1 Functional Forms

The **Viability Potential** of a task $\mathbf{x}$ at time t is defined as $\Phi(\mathbf{x}, t) = p(\mathbf{x}, t)$, where

- $p(\mathbf{x}, t)$ is the per-unit revenue generated by performing the task.

The **Viability Threshold** of a task $\mathbf{x}$ at time t, is defined as $\Phi^{min}(\mathbf{x}, t) = c_v(\mathbf{x}, t) + c_f(\mathbf{x}, t)$, where

- $c_v(\mathbf{x}, t) = \min\left\{\frac{w(\mathbf{x},t)}{a^L(\mathbf{x},t)}, \frac{r(\mathbf{x},t)}{a^K(\mathbf{x},t)}\right\}$ is the minimized per-unit variable cost of performing the task using the optimal task factor (labor or capital), and
- $c_f(\mathbf{x}, t)$ is the allocated per-unit fixed cost of performing the task.

The **Viable Subset** $E(t) = \{\mathbf{x} \in \mathcal{T} \mid p(\mathbf{x}, t) \geq c_v(\mathbf{x}, t) + c_f(\mathbf{x}, t)\}$ thus contains the set of tasks whose costs do not exceed their revenues, i.e., the tasks generating non-negative profits.

The **Automation Potential** of a task $\mathbf{x}$ at time t is defined as $\Psi(\mathbf{x}, t) = \frac{a^K(\mathbf{x},t)}{r(\mathbf{x},t)}$, where

- $a^K(\mathbf{x}, t)$ is the task's capital productivity, and
- $r(\mathbf{x}, t)$ is the task's cost of capital.

The **Automation Threshold** of a task $\mathbf{x}$ at time t is defined as $\Psi^{min}(\mathbf{x}, t) = \frac{a^L(\mathbf{x},t)}{w(\mathbf{x},t)}$, where

- $a^L(\mathbf{x}, t)$ is the task's labor productivity, and
- $w(\mathbf{x}, t)$ is the task's wage rate of labor.

The **Automatable Subset** $A(t) = \{\mathbf{x} \in \mathcal{T} \mid \frac{a^K(\mathbf{x},t)}{r(\mathbf{x},t)} \geq \frac{a^L(\mathbf{x},t)}{w(\mathbf{x},t)}\}$ thus contains tasks where capital is at least as effective as labor on a cost-adjusted basis. This condition arises directly from the firm's cost-minimization problem; the decision to automate compares the unit cost of production using each factor: capital is chosen if $\frac{r(\mathbf{x},t)}{a^K(\mathbf{x},t)} \leq \frac{w(\mathbf{x},t)}{a^L(\mathbf{x},t)}$, corresponding to the condition $\Psi(\mathbf{x}, t) \geq \Psi^{min}(\mathbf{x}, t)$.

The **Task Magnitude**, measuring the amount of task $\mathbf{x}$ performed at time t, is defined as $M(\mathbf{x}, t) = q^*(\mathbf{x}, t)$, where $q^*(\mathbf{x}, t)$ is the task's equilibrium quantity. In this interpretation, each existing task in the viable subset $E(t)$ has a positive equilibrium quantity:

$$M(\mathbf{x},t) = \begin{cases} q^*(\mathbf{x},t), & \mathbf{x} \in E(t), \\ 0, & \mathbf{x} \notin E(t) \end{cases}$$

The **Labor Task Share**, corresponding to the *non-automated task share* defined in Section 3.1, is now formulated as:

$$\Gamma(t) = \frac{\int_{E(t)\setminus A(t)} q^*(\mathbf{x},t)\, d\mathbf{x}}{\int_{E(t)} q^*(\mathbf{x},t)\, d\mathbf{x}},$$

whose properties are further explored on the aggregate level in Section 5.2

With these functional forms, the viable subset and automatable subset are related; shocks to factor prices and productivities simultaneously impact both subsets' component functions, implying changes in technology and input costs jointly alter the tasks that exist and the tasks that are automated.

Although we define the viability and automation functions as current-period objects, they can be interpreted heuristically as *discounted expectations of future revenues, costs, and factor productivities.* In this interpretation, $E(t)$ and $A(t)$ can encompass tasks that are unprofitable or not yet automatable, with current-period forms expected to cross their respective thresholds in the future.

4.2 Economic Parameters and Dynamics

The dynamics of the viable subset $E(t)$ and automatable subset $A(t)$, as outlined by the dynamics in Section 3.2, are now interpreted by changes in underlying economic parameters defining the component functions:

- Changes in task revenue, variable costs (determined by factor productivities and factor prices), or allocated fixed costs drive operational viability and thus the size/shape of $E(t)$.
- Changes in factor productivities and factor prices drive relative cost-effectiveness between capital and labor, directly determining the size/shape of A(t).

Corresponding to the boundary dynamics discussed in Section 3.2, we now interpret the expansion, contraction, or stasis of the viable subset $E(t)$ and the automatable subset $A(t)$.

Dynamics of the Viable Subset:

- **Task Instantiation:** A task $\mathbf{x}$ is *instantiated* when its magnitude flips from $M(\mathbf{x}, t-\epsilon) = 0$ to $M(\mathbf{x}, t) > 0$ while the profit condition $p(\mathbf{x}, t) \geq c_v(\mathbf{x}, t) + c_f(\mathbf{x}, t)$ already holds. Instantiation can occur anywhere inside $E(t)$; if $\mathbf{x} \notin E(t-\epsilon)$ the boundary simply expands at the *same* moment, but that expansion is not what defines the new task.
- **Task Obsolescence:** A task is no longer performed when its magnitude falls to zero, $M(\mathbf{x}, t) = 0$. This may be because it turns unprofitable ($e(\mathbf{x}; t) < 0$) *or* because demand evaporates even though the task remains viable.
- **Task Stasis:** A task persists when it continues to be performed with $M(\mathbf{x}, t) > 0$ and $e(\mathbf{x}; t) \geq 0$.

Dynamics of the Automatable Subset:

- **New Automation:** A task $\mathbf{x}$ becomes automatable when capital's cost-adjusted productivity rises to meet or exceed labor's cost-adjusted productivity, i.e., when $\frac{a^K(\mathbf{x},t)}{r(\mathbf{x},t)}$ rises to satisfy $\frac{a^K(\mathbf{x},t)}{r(\mathbf{x},t)} \geq \frac{a^L(\mathbf{x},t)}{w(\mathbf{x},t)}$. This corresponds to $a(\mathbf{x}; t) \geq 0$, occurring at boundaries where $\frac{\partial a}{\partial t} > 0$. This can result from capital-augmenting technological improvements increasing $a^K(\mathbf{x}, t)$, decreases in capital costs $r(\mathbf{x}, t)$, increases in labor costs $w(\mathbf{x}, t)$, or decreases in labor productivity $a^L(\mathbf{x}, t)$.
- **De-Automation:** A task $\mathbf{x}$ becomes non-automatable when its capital cost-adjusted productivity falls below its labor cost-adjusted productivity, i.e., $\frac{a^K(\mathbf{x},t)}{r(\mathbf{x},t)} < \frac{a^L(\mathbf{x},t)}{w(\mathbf{x},t)}$. This corresponds to $a(\mathbf{x}; t) < 0$, occurring at boundaries where $\frac{\partial a}{\partial t} < 0$. This can result from labor-augmenting technological improvements increasing $a^L(\mathbf{x}, t)$, decreases in labor costs $w(\mathbf{x}, t)$, increases in capital costs $r(\mathbf{x}, t)$, or decreases in capital productivity $a^K(\mathbf{x}, t)$.

- **Automation Stasis:** A task $\mathbf{x}$ remains in the automatable subset $A(t)$ as long as it satisfies the factor productivity condition $\frac{a^K(\mathbf{x},t)}{r(\mathbf{x},t)} \geq \frac{a^L(\mathbf{x},t)}{w(\mathbf{x},t)}$. It must remain relatively more efficient to perform with capital.

The net impact on the labor task share, and thus labor income share (see Section 5.2), depends on the specific trajectories of the viable and automatable subsets across task space.

5 Integrating A&R

We now apply the theory developed in Section 3 and specific functional forms of Section 4 to bridge towards the task-based framework. We first interpret the foundational A&R concepts in task space, then map out microfoundations for the aggregate model.

5.1 Foundational Concepts

The core concepts of the A&R model map naturally to the task space model:

- **Automation:** An increase in the set of automated tasks $I \to I'$ within the task continuum $[N-1, N]$ corresponds to the expansion of the automatable subset $A(t)$ within the viable subset $E(t)$. Specifically, it occurs when $a(\mathbf{x};t)$ becomes non-negative for viable tasks $\mathbf{x} \in E(t)$ that were previously non-automated.

- **Displacement Effect:** When $[N-1, N]$ is fixed, an increase in the set of automated tasks $I \to I'$ decreases the labor task share $\Gamma(I, N)$ as more tasks are performed by capital. This corresponds to the expansion of the automation boundary $a(\mathbf{x};t)$ within a fixed E, converting non-automated tasks in $\mathbf{x} \in E \setminus A(t)$ into automated tasks in $\mathbf{x} \in E \cap A(t)$, which likewise reduces $\Gamma(t)$.

- **New Tasks:** The creation of new tasks $N \to N'$ in A&R corresponds in our model to either viable tasks $\mathbf{x} \in E(t)$ transitioning from not being performed to being performed, i.e., $M(\mathbf{x}, t-\epsilon) = 0 \to M(\mathbf{x}, t) > 0$, or previously non-viable tasks becoming viable and performed as the viable subset expands, i.e., $\mathbf{x} \notin E(t-\epsilon) \rightarrow \mathbf{x} \in E(t)$ with $M(\mathbf{x}, t) > 0$.

- **Reinstatement Effect:** When I is fixed in A&R, new tasks $N \rightarrow N'$ increase the labor task share $\Gamma(I, N)$, as they are assumed to be initially performed by labor. In our model, this corresponds to either previously unperformed tasks $\mathbf{x}$ within $E(t) \setminus A(t)$ beginning to be performed with positive magnitude, or new tasks entering the viable non-automatable region as the viable subset expands into non-automatable areas, i.e., $\mathbf{x} \notin E(t-\epsilon) \rightarrow \mathbf{x} \in E(t) \setminus A(t)$ with $M(\mathbf{x}, t) > 0$. Both mechanisms increase the labor task share $\Gamma(t)$.

Building on A&R's discrete task boundaries N and I in one dimension, the task space framework enables n-dimensional analysis of the non-uniform growth of $E(t)$ and $A(t)$ and their boundary mechanics.

5.2 Synthesizing the Frameworks

Connecting Micro and Macro Variables: The task-specific parameters defined in Section 4 aggregate to economy-wide variables that appear in the A&R model. We express these aggregate and magnitude-weighted average variables as follows:

- Aggregate labor input: $L(t) = \int_{E(t)\setminus A(t)} q^*(\mathbf{x}, t)\, d\mathbf{x}$,
- Aggregate capital input: $K(t) = \int_{E(t)\cap A(t)} q^*(\mathbf{x}, t)\, d\mathbf{x}$,
- Average wage: $W(t) = \frac{\int_{E(t)\setminus A(t)} w(\mathbf{x},t) q^*(\mathbf{x},t)\, d\mathbf{x}}{\int_{E(t)\setminus A(t)} q^*(\mathbf{x},t)\, d\mathbf{x}}$
- Average rental rate: $R(t) = \frac{\int_{E(t)\cap A(t)} r(\mathbf{x},t) q^*(\mathbf{x},t)\, d\mathbf{x}}{\int_{E(t)\cap A(t)} q^*(\mathbf{x},t)\, d\mathbf{x}}$
- Average labor productivity: $A^L(t) = \frac{\int_{E(t)\setminus A(t)} a^L(\mathbf{x},t) q^*(\mathbf{x},t)\, d\mathbf{x}}{\int_{E(t)\setminus A(t)} q^*(\mathbf{x},t)\, d\mathbf{x}}$
- Average capital productivity: $A^K(t) = \frac{\int_{E(t)\cap A(t)} a^K(\mathbf{x},t) q^*(\mathbf{x},t)\, d\mathbf{x}}{\int_{E(t)\cap A(t)} q^*(\mathbf{x},t)\, d\mathbf{x}}$

Production Function: Using the aggregate variables defined above, final output $Y(t)$ can be expressed as a Constant Elasticity of Substitution (CES) production function over tasks performed by labor and capital, following A&R:

$$Y(t) = \Pi(t) \left(\Gamma(t)^{\frac{1}{\sigma}} \left(A^L(t)L(t) \right)^{\frac{\sigma-1}{\sigma}} + (1-\Gamma(t))^{\frac{1}{\sigma}} \left(A^K(t)K(t) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where

- $Y(t)$ is aggregate output at time t,
- $L(t)$ and $K(t)$ are aggregate labor and capital inputs at time t,
- $A^L(t)$, $A^K(t)$ are factor-augmenting technology terms for labor and capital,
- σ is the constant elasticity of substitution between capital and labor aggregates,
- $\Pi(t)$ is productivity gains from changes in the range of tasks, and
- $\Gamma(t)$ is the labor task share of production at time t.

In the A&R framework, the labor task share is represented by the fraction of tasks indexed greater than I in $[N-1, N]$. In our framework, it is represented by the fraction of tasks in $E(t)$ that are not in $A(t)$.

Subsequently, the labor income share is given by:

$$s^L = \frac{1}{1 + \frac{1-\Gamma(t)}{\Gamma(t)} \left(\frac{R(t)/A^K(t)}{W(t)/A^L(t)} \right)^{1-\sigma}},$$

where $W(t)$ and $R(t)$ are the aggregate wage and rental rates defined above. This equation highlights how the aggregate labor income share s^L depends on the relation between the task allocation (captured by Γ, determined by the relative boundaries of $E(t)$ and $A(t)$ in our model) and the relative effective factor prices ($\frac{R(t)/A^K(t)}{W(t)/A^L(t)}$), which aggregate from the underlying task-specific productivities ($a^K(\mathbf{x}, t)$ and $a^L(\mathbf{x}, t)$) and factor costs ($r(\mathbf{x}, t)$ and $w(\mathbf{x}, t)$) defined in Section 4.

Our model thus provides an analytical framework for describing labor outcomes, deriving it from task-specific profitability and automation decisions based on underlying economic parameters. By endogenizing $\Gamma(t)$ based on these micro-level factors, the model directly connects task-specific technological changes to the aggregate labor income share s^L, providing a formal explanation for observed trends in income distribution.

6 Visualization in Two Dimensions

The task space model can be used to visualize how human- and machine-performed tasks evolve over time. This section presents a stylized two-dimensional representation of the model, with axes representing routine-to-non-routine (x-axis) and manual-to-cognitive (y-axis) task attributes. These visualizations are not data-driven but rather conceptual illustrations of how the viable subset $E(t)$ and automatable subset $A(t)$ change across four different historical periods and two potential future scenarios.

In these visualizations, viable non-automatable tasks $E(t) \setminus A(t)$ are displayed in light green, viable automatable tasks $E(t) \cap A(t)$ are displayed in dark blue, and non-viable, automatable tasks $A(t) \setminus E(t)$ are displayed in light blue. The progression of these regions demonstrates both the expansion of economically viable tasks and automation capabilities over time. Non-viable, non-automatable regions outside $A(t) \cup E(t)$ are represented as a blank white background to denote inactivity.

These representations contain non-convex geometries rather than a smooth convex frontier. Mathematically, it results from several overlapping surfaces that cross and fold, so when we compress the boundary into two dimensions it breaks into scattered pieces with gaps in between. Empirically, each new technology or cost shift opens up small clusters of viable or automatable tasks.

6.1 Human History in Four Graphs of Task Space

Figure 1: Early Agricultural Society

The first visualization depicts task space in early agricultural society, characterized by a limited viable subset concentrated in routine-manual task space.

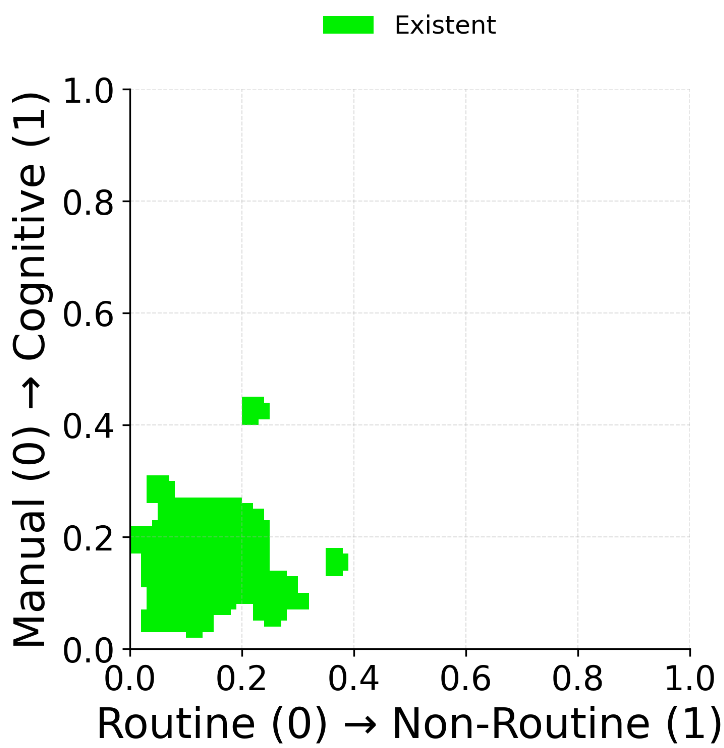

Figure 1: Task Space in Early Agricultural Society

This represents primarily agricultural tasks, with scattered isolated regions representing specialized roles such as merchants, artisans, and governance. The absence of automation reflects pre-industrial technological capabilities, with all viable tasks performed by human labor.

Figure 2: Early Industrial Revolution (c. 1800)

The second visualization shows the initial impact of industrialization, with the viable subset expanding notably and the first pockets of automation emerging.

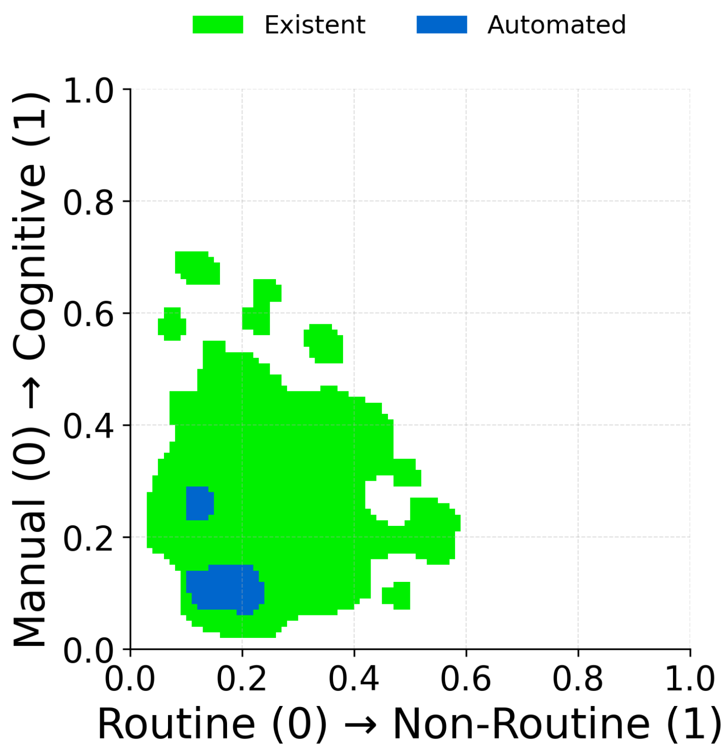

Figure 2: Task Space in the Early Industrial Revolution

These automated regions correspond to early mechanization through innovations like the printing press, steam engine, and mechanical looms. These technologies primarily automated routine manual tasks, establishing the initial pattern of automation targeting tasks with high routine components.

Figure 3: Mid-20th Century (c. 1960)

By the mid-20th century, the viable subset has grown substantially, encompassing a much wider range of tasks across both routine and non-routine dimensions.

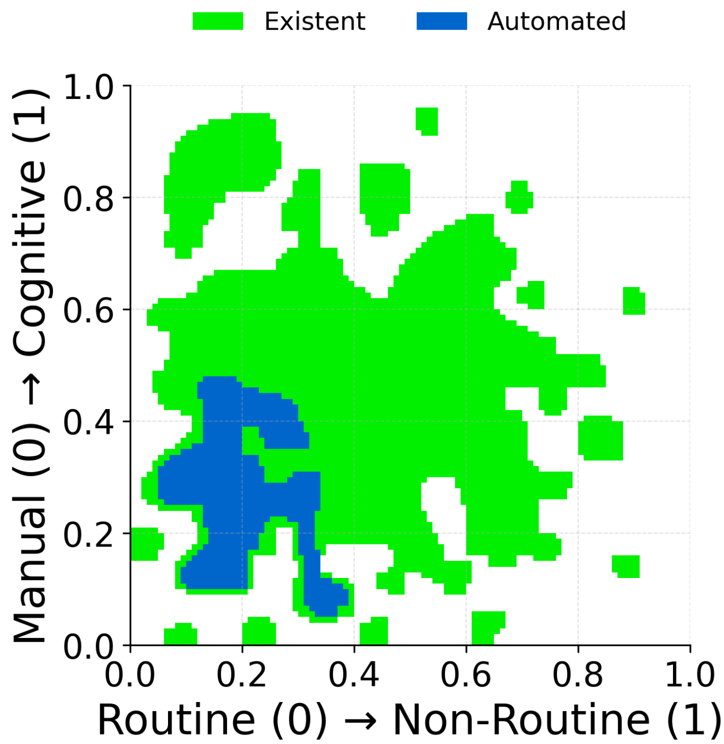

Figure 3: Task Space in the Mid-20th Century

Automation has expanded significantly, covering most routine manual tasks through advanced mechanization in factories (assembly lines, specialized machinery) and early automation of routine cognitive tasks through electromechanical computing devices and early computers. The boundary of automation now pushes into the cognitive dimension but remains largely confined to routine tasks.

Figure 4: Early Digital Era (c. 2000)

This visualization captures the impact of the digital revolution and skill-biased technological change (SBTC).

Software and computerization have automated many routine cognitive tasks (ac-

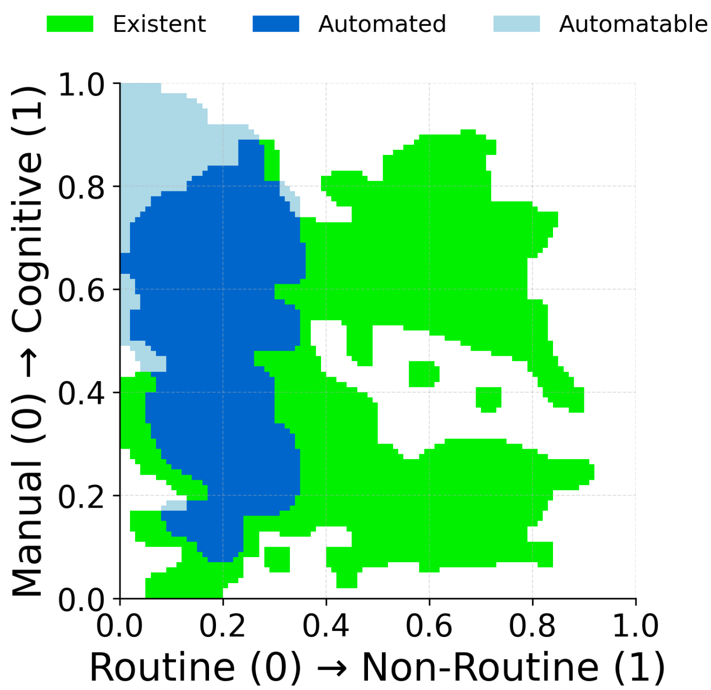

Figure 4: Task Space in the Early Digital Era

counting, information processing, basic data analysis). A new feature appears in this era: the light blue region of technically automatable but economically non-viable tasks, representing capabilities enabled by software but not yet economically implemented at scale.

6.2 Two Possible Futures in Task Space

Figure 5: AI Dominant, Limited Robotics (c. 2040)

This first future scenario depicts an economy where large language models and AI capabilities have continued to advance, pushing automation deep into the non-routine cognitive space.

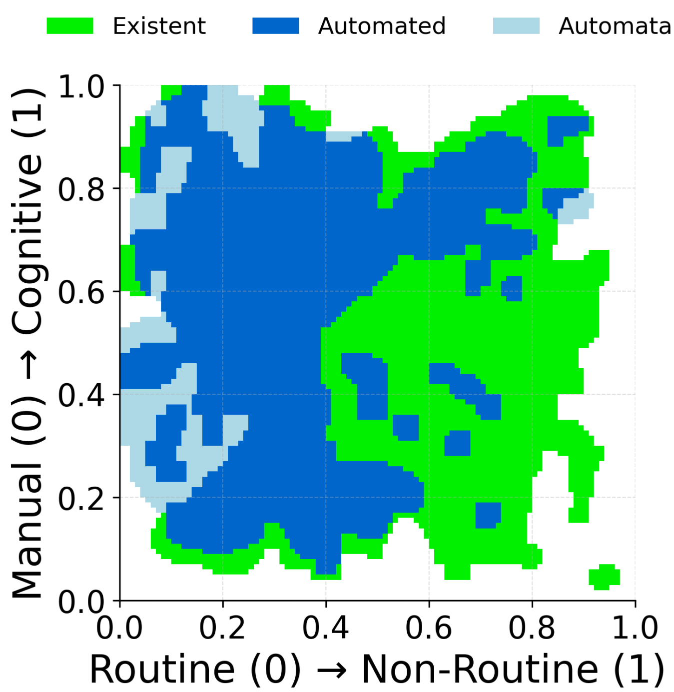

Figure 5: Predicted Task Space with Advanced AI, Limited Robotics

Tasks such as content creation, analysis, legal research, and medical diagnostics have become largely automated as the automation boundary has expanded significantly in the cognitive dimension. However, robotics development has been constrained by higher costs and technical limitations, leaving most non-routine manual tasks (like skilled trades, elder care, and complex physical manipulation) relatively untouched by automation.

Figure 6: Comprehensive Automation (c. 2040) This second future scenario shows significant advances in both AI and robotics, with automation extending across nearly the entire viable subset.

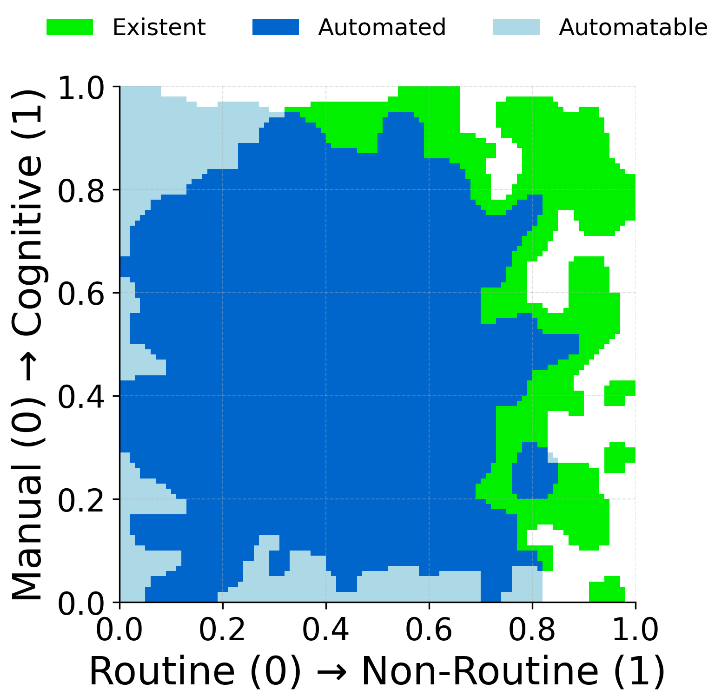

Figure 6: Predicted Task Space with Advanced AI and Robotics

Advanced general-purpose AI systems have mastered most cognitive tasks, while robotics breakthroughs have enabled automation of complex manual tasks through developments in dexterity, sensory capabilities, and adaptability. In this scenario, the few remaining non-automated tasks are at the extreme edges of non-routine complexity or require uniquely human capabilities that remain economically advantageous.

6.3 Summary

These visualizations illustrate the key dynamics of the task space model: the expansion of the viable and automatable subsets through economic development and technological progress. They highlight how automation has historically targeted routine tasks first before expanding into non-routine domains, and how the boundaries between human and machine tasks have shifted over time. The future scenarios demonstrate alternative paths for how AI and robotics might reshape these boundaries, with significant implications for labor markets and income distribution. The visualization also demonstrates how task proximity correlates with similar automation vulnerability, as adjacent tasks in our two-dimensional representation tend to become automated during similar technological epochs.

7 Future Avenues & Conclusion

7.1 Mathematical Formalization

Boundary Dynamics: The viability and automation boundaries exhibit complex dynamic properties under varying technological growth rates. We can derive conditions for different boundary behaviors including monotonic expansion, oscillatory transitions, and discontinuous jumps by developing differential equations governing the time derivatives of difference functions.

Topological Properties: Under varying assumptions about the potential and threshold functions, we can establish general properties regarding the homotopy type, genus, and boundary complexity of these sets as they evolve through time. These properties determine whether automation progresses through contiguous expansion, topological merging of previously disconnected components, or the formation of complex boundaries between the viable and automatable subsets.

7.2 Economic Model Integration

Income Distribution: The mapping between worker skill distributions and task space regions yields precise predictions about wage inequality dynamics under technological change. When worker skills cluster near automation boundaries, even marginal technological shocks can produce significant shifts in income distribution, which can explain historical wage polarization observed in developed economies or predict future income distributions.

Task Bundling: Occupations are composed of multiple complementary tasks, indicating partial automation of task subsets could transform job requirements rather than eliminating positions entirely. The properties of these bundles determine whether technological advances lead to job polarization, upskilling, or wholesale displacement, with implications for occupational mobility and retraining requirements. Future work could apply insights from Bittarello, Kramarz, & Maitre [8], who explore how tasks bundle into occupations with varying automation risks.

General Equilibrium Effects: Firm-level automation decisions generate feedback effects through factor markets, as the displacement of labor in one region of task space alters relative factor prices economy-wide. This relationship between automation decisions and factor prices creates potential multiple equilibria and path dependencies, where initial conditions significantly influence long-run task allocation outcomes.

Endogenous Automation: Innovation investment decisions systematically direct technological progress toward specific regions of task space based on expected returns. When modeled as optimal control problems with resource constraints, these decisions reveal how market incentives, institutional factors, and public R&D allocation shape not just the pace but the direction of automation in task attribute space.

7.3 Empirical Testing

The empirical validation of the task space model requires constructing a multidimensional attribute representation of an economy's task space through a combination of occupational databases and natural language processing techniques.

Using O*NET task descriptions, skill requirements from job postings, and historical technological adoption data, we can estimate the coordinates of tasks in attribute space and track the temporal evolution of the viability and automation boundaries across different economic sectors. Machine learning algorithms applied to these datasets can identify the implicit dimensionality of the task space and reveal which attribute combinations most strongly predict automation vulnerability.

Additionally, panel data on occupation-specific wage premiums, employment shares, and capital-labor substitution rates can be used to calibrate the potential and threshold functions, while analysis of historical technological shocks enables causal identification of displacement effects. Cross-country variation in factor prices and technology adoption provides additional identification leverage, allowing us to distinguish between task attributes that are universally susceptible to automation and those whose vulnerability depends on local economic conditions.

References

[1] Daron Acemoglu. Directed Technical Change. *The Review of Economic Studies*, 69(4):781–809, 10 2002.

[2] Daron Acemoglu. The Simple Macroeconomics of AI. *Economic Policy*, 40(121):13–58, January 2025. Published online: 13 May 2024.

[3] Daron Acemoglu and David H. Autor. Skills, Tasks and Technologies: Implications for Employment and Earnings. In Orley Ashenfelter and David Card, editors, *Handbook of Labor Economics*, volume 4, chapter 12, pages 1043–1171. Elsevier, 2011.

[4] Daron Acemoglu, Fredric Kong, and Pascual Restrepo. Tasks At Work: Comparative Advantage, Technology and Labor Demand. Working Paper 32872, National Bureau of Economic Research, August 2024.

[5] Daron Acemoglu and Jonas Loebbing. Automation and Polarization. Working Paper 30528, National Bureau of Economic Research, September 2022.

[6] Daron Acemoglu and Pascual Restrepo. Automation and New Tasks: How Technology Displaces and Reinstates Labor. *Journal of Economic Perspectives*, 33(2):3–30, Spring 2019.

[7] David H. Autor, Frank Levy, and Richard J. Murnane. The Skill Content of Recent Technological Change: An Empirical Exploration. *The Quarterly Journal of Economics*, 118(4):1279–1333, 2003.

[8] Luca Bittarello, Francis Kramarz, and Alexis Maitre. The task content of occupations. *Revue économique*, 75(1):pp. 31–54, 2024.

[9] Kelvin J. Lancaster. A New Approach to Consumer Theory. *Journal of Political Economy*, 74(2):132–157, 1966.

[10] Ilse Lindenlaub. Sorting Multidimensional Types. *Econometrica*, 85(1):287–326, 2017.

[11] Pascual Restrepo. Automation: Theory, Evidence, and Outlook. *Journal of Economic Literature*, 62(1):3–69, March 2024.

[12] Sherwin Rosen. Hedonic Prices and Implicit Markets: Product Differentiation in Pure Competition. *Journal of Political Economy*, 82(1):34–55, 1974.

[13] Joseph Zeira. Workers, Machines, and Economic Growth. *The Quarterly Journal of Economics*, 113(4):1091–1117, 1998.