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UNDERGRADUATE RESEARCH FELLOWSHIP WORKING PAPER

The MICAH Model of War (Modified Ising Configuration of Armed Hostilities)

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Abstract

Borrowing the 2D Ising Model from condensed matter physics and applying its framework to war, we demonstrate how a four-tiered complex game of war is modeled using the Ising lattice (with a supplemental modified SIR Model to demonstrate troop dynamics). Since hierarchically coupled games can bottom-to-top be simulated by the Ising Model without oversimplifying the complexity that produces emergent phenomenon, this research functions as a crude attempt to describe hierarchically coupled games in their proper complexity and motivate a deeper model of understanding war. Mapping the Ising Model – extensively studied in physics and readily modeled computationally – over to conflict scenarios provides economists and strategists a novel simulation environment to approach hierarchically coupled games.

Introduction and Motivation

As the 19th century Prussian field marshal Helmuth von Moltke once famously quipped, "No plan survives first contact with the enemy" (1871). So how does a general act in a dynamic system like war? How do his subordinate officers and enlisted personnel make decisions and what impact do they have on the war at large? Traditional game theory struggles to *visually* depict bottom-to-top the layered interplay within complex games such as war, with existing paradigms serving as mathematical frameworks which are difficult to graphically depict at full scale. Rather than attempting to reduce a real-world phenomenon into a reductive game theory skeleton – missing intuitive nuance in the process – this paper serves as a crude attempt to motivate an alternative visualization of the game of war in its proper complexity. On the tactical level, soldiers play a simultaneous game commanded by sergeants playing a sequential game. This pairing is interrelated with the strategic game simultaneously transpiring, consisting of a major's simultaneous game nestled within a general's sequential game. Each rank within the game possesses its own utility function, its own goals and ambition, and its own distinct dynamics. When a major makes a move, for instance, he relies upon a confluence of factors to determine his play – his general's importance weighting for this territory, his sergeants' reports from in-the-field, and his own experience, training, and biases. Typically, these games are simplified for analysis' sake, but intuition to visualize is lost in the simplification.

To evaluate these games – and attempt to formulate the beginnings of an answer to these inquiries – we computationally employ the 2D Ising Model from condensed matter physics. The Ising Model provides a perfect visualization to observe how the different games progress at different levels. Intuitively, by observing how the Ising lattice iterates through time, we computationally simulate how combat evolves, incorporating chaos and probabilities, which are intrinsically baked into the Ising model. Randomness in combat outcomes (affected by strategic decisions, but ultimately probabilistic) is tantamount to randomness in spin orientation in the Ising model, thereby equating the outcome of strategic decisions to an atom's spin orientation since both are comparably arbitrary but affected by deterministic decisions. From the Ising framework, we overlay a modified SIR (Susceptible-Infected-Removed) Model from epidemiology to track troop dynamics across the lattice (Prodanov, 2022).. The 2D Ising Model assists in describing and visualizing how such phenomenon and unexpected eventualities propagate across levels of a hierarchically coupled game via importance weightings. Considering how the actions (and perceptions) of one level's game affects the coupled counterparts, the general's and major's processing of battlefield information passed up from their subordinates – combined with their training, experience, and beliefs – sets regional importance weightings for various wartime objectives, establishing the strategic agenda. In turn, these regional importance weightings back-propagate, influencing battlefield realities at the local (i.e. soldier and sergeant) level and dictating the course of the war in the trenches.

The 2-Dimensional Ising Model

First, consider the 2D Ising Model. For a rigorous physics definition of the 2D Ising Model, reference Onsager (1944) and Yeomans (1992), but consider a conceptual summary here. In condensed matter physics, the Ising Model is utilized to describe magnetic behavior of materials, specifically the spin behavior of atoms as the material they comprise phase transitions from a ferromagnetic to paramagnetic state. To understand the Ising Model, picture a 2-dimensional lattice (aka grid) of periodically spaced atoms which constitute a solid object, such as a thin sheet of iron (see Figure 1). Each lattice site (i.e., atom) interacts with its four nearest neighbors (up, down, left, and right) and possesses a binary spin variable which assumes either a +1 (spin up) or –1 (spin down) value. The summation of a lattice's spins dictates its overall magnetic properties, known as "net magnetization" (i.e., whether it is ferromagnetic or paramagnetic). In the generalized formulation of the 2D Ising Model, the spins' magnitude can assume any positive or negative integer value.

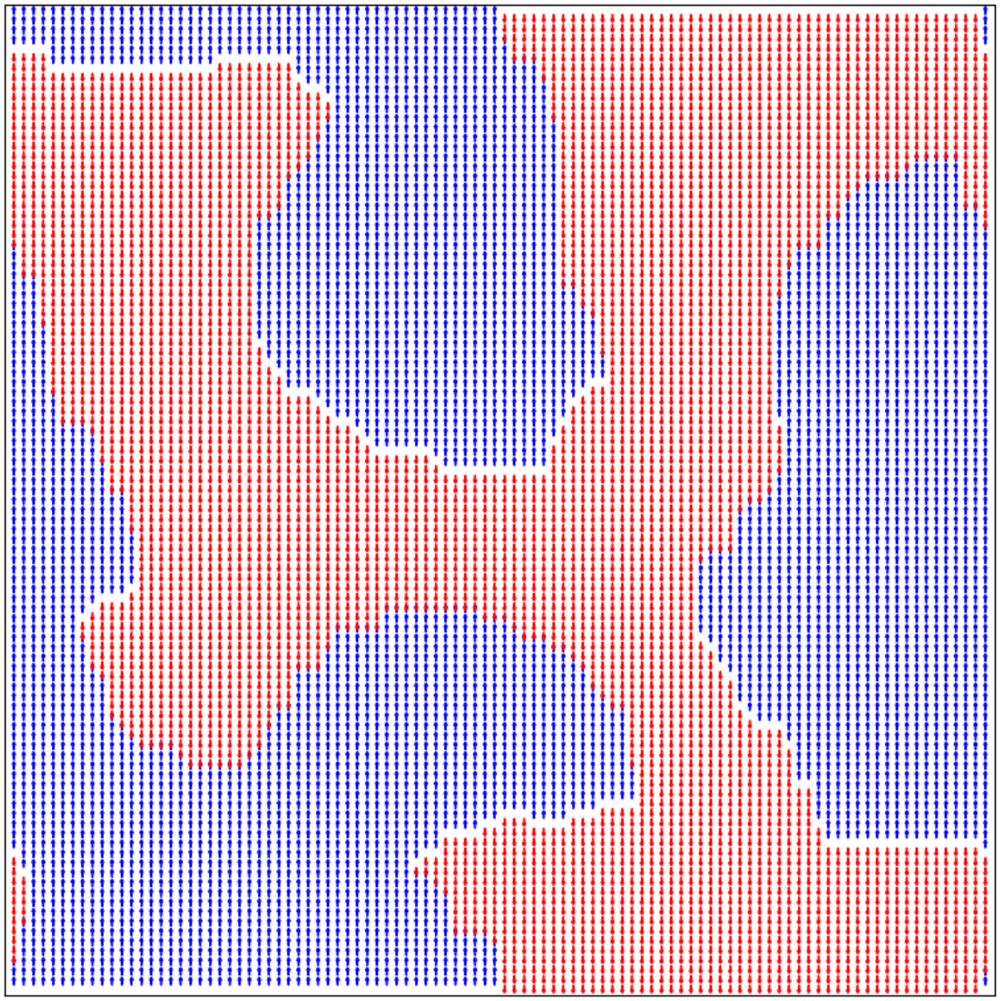

Figure 1: Perfectly ordered domains (i.e., at absolute zero temperature) in a 2D Ising lattice. Only realizable at the start of a war. Generated via author's Python program.

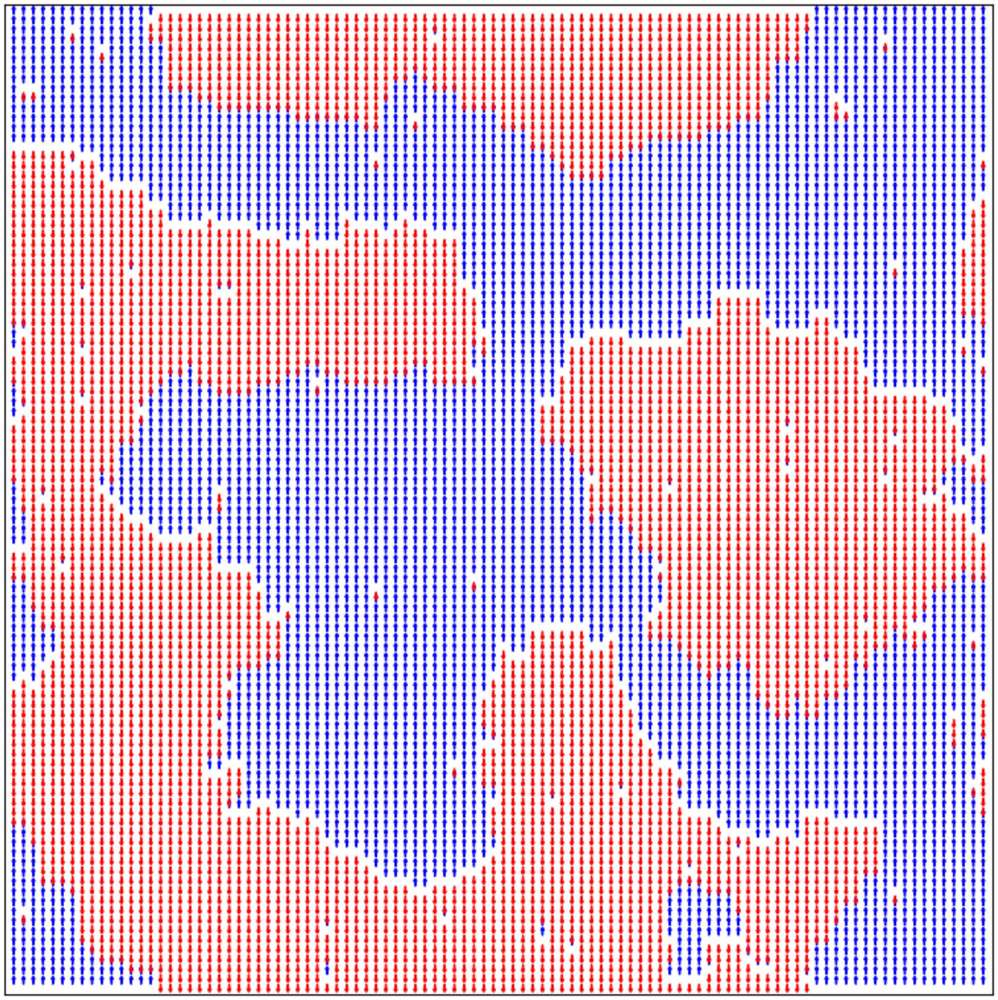

Figure 2: Imperfections arise in ordered domains (above absolute zero; the primary regime of interest for this paper). Generated via author's Python program.

Starting from absolute zero, ferromagnetic materials contain ordered "domains" (i.e., zones) where the lattice sites maintain uniform spins of either exclusively "up" or exclusively "down" (Figure 1), with their spins correlated across the domain. As temperature increases (Figures 2 and 3), imperfections arise in these domains (i.e., isolated spin up arrows emerge in a spin down domain), until the critical temperature (Figure 4), where the domains disintegrate altogether. Figures 1-4 represent this progression, as temperature increases. At the critical temperature, the correlation distance diverges, meaning that spins are correlated across the entire lattice rather than just within a specific domain. Interestingly, a recursive fractal pattern emerges as the ferromagnetic domains collapse. Above the critical temperature (Figure 4), the correlation between neighboring atoms shrinks dramatically, so that only the spins of a site's direct neighbors affect that atom's spin orientation. When a material is approaching the critical temperature from either direction, a property known as "magnetic susceptibility" explodes towards infinity. By placing a small external magnet beside the lattice when its magnetic susceptibility is high (i.e., when it is nearing its critical temperature), one can align virtually *all* of the lattice's spins, thereby turning the lattice into an *extremely powerful magnet.* The lattice material's hysteresis curve dictates how long all the spins stay aligned until they entropically deteriorate back into their original ferromagnetic domains (if below the critical temperature) or paramagnetic chaos (if above). Refer to Appendix for Python code (author's original work coded via AI assistance) whose simulation rendered these diagrams.

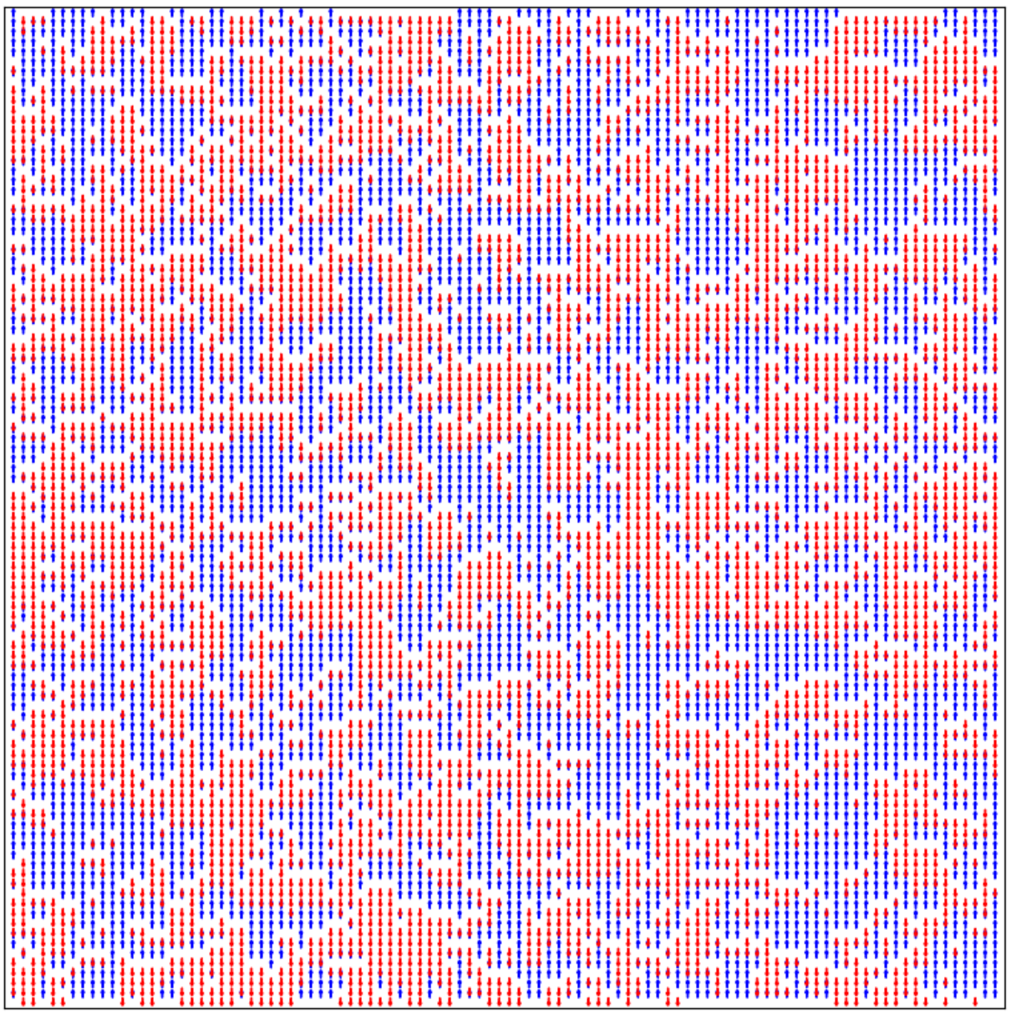

Figure 3: Continuing to increase temperature, but still below critical temperature, domains are remain somewhat recognizable, but are rapidly breaking down. Generated via author's Python program.

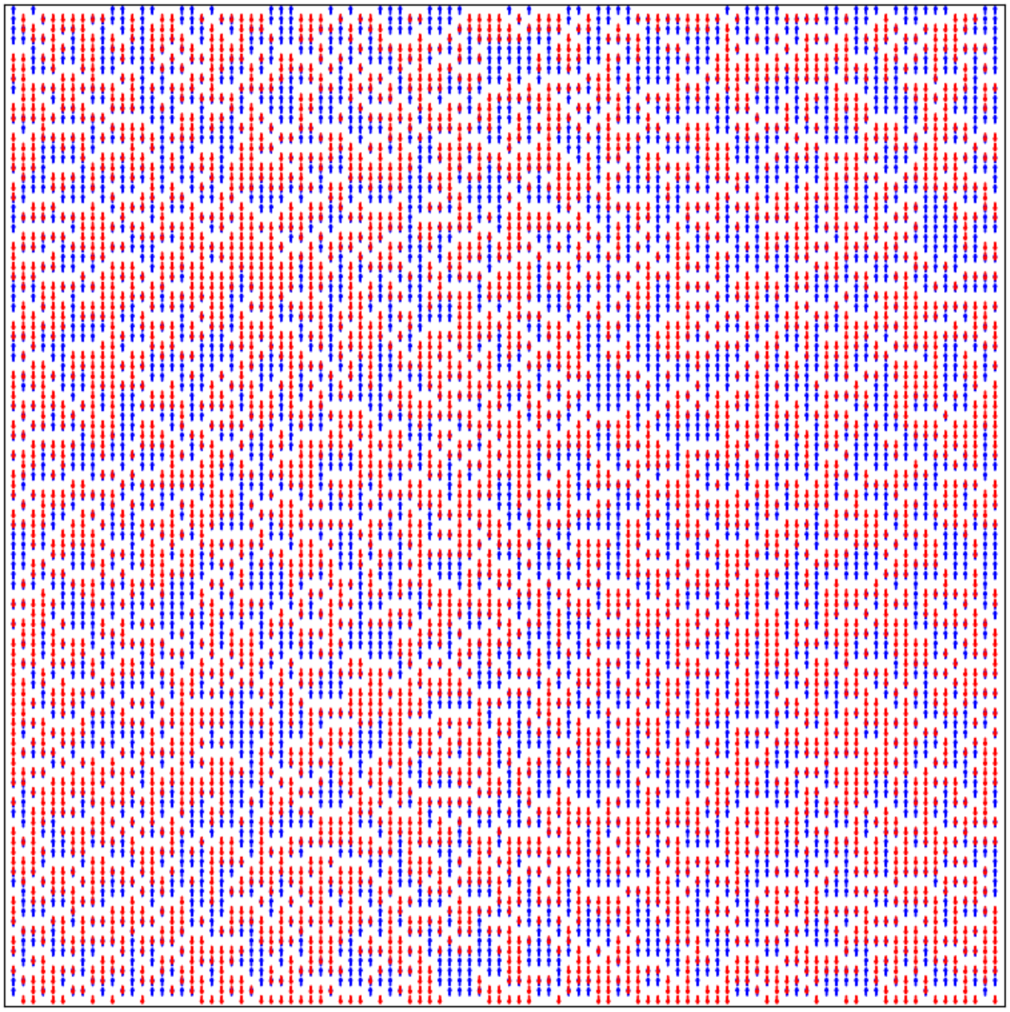

Figure 4: Above critical temperature, domains vanish altogether, limiting the lattice to localized-effects only. Generated via author's Python program.

Mapping A Hierarchically Coupled Game (War) onto the 2D Ising Model

To model war, picture a 2-dimensional lattice of arrows (Figure 1). Within the lattice, there exist various domains, which are controlled by either red or blue forces, with varying degrees of security. For instance, a heavily-blue domain may contain some imperfections (regions held by red forces), despite the fact that the overall domain is still solidly under blue's control (see Figure 2). Each domain contains a number of arrows or lattice sites, which correspond to tactical objectives on the battlefield, which can be controlled by either blue or red forces. For the purposes of this paper, tactical objectives range from airports and radio stations to hilltop vantage points and river crossings. Tactical objectives are represented by arrows, with orientation (i.e., spin) either up or down, corresponding to friendly or enemy control respectively. The magnitude of the up/down arrow corresponds to how intensely red or blue controls that particular tactical objective. For instance, an up arrow of magnitude 100 is far more securely under blue's influence than an up arrow of magnitude 5 (Note: the magnitudes in Figures 1-4 do not range, and assume a binary value of either +1 or –1). Tactical objectives vary in importance – and this importance dynamically shifts (dictated by the major's importance factor, computed via a combination of his superior's orders, subordinates' reports, and his own training/perspective) as the battlefield evolves. A hilltop once deemed irrelevant may now become the focal point of the domain's struggle. Entire domains' importance – set by generals – also shifts as the war evolves. Additionally, neighboring tactical objectives border each other on the lattice but can vary in real-world distance from each other. Crucially, they are still in the vicinity of one another (i.e., events at one tactical objective affect events at the neighboring objectives), which limits their spacing to relatively close to each other. Unexpected results and eventualities arise at every level of this game, and the 2D Ising Model – with its built-in capacity to simulate order out of randomness – accounts for it, demonstrating how the levels of the hierarchically coupled game adjust and iterate according to the perturbation. For uniformity, the command structure of the U.S. Army is employed throughout this paper (PBS, 2022), with a general commanding ~25 majors, who each command ~16 sergeants, who in turn herd ~10 soldiers. In the actual military, more than these 4 ranks and classifications exist, but for the sake of brevity and simplicity, anytime a subgame exists (i.e., the colonels under a general), it is automatically condensed into its larger game. In our modeling, generals determine troop distribute from domain to domain (via an importance weighting which they select) and issue orders to majors, who determine troop distribution within a given domain (via an importance weighting which they select) and command sergeants, who choose which tactics to employ and command their soldiers, who are essentially are essentially limited to choices which preserve their lives so that they can "shoot before being shot". Obviously, there is a finite quantity of troops (and corresponding war materiel) to distribute within any given lattice/domain/lattice site.

In mapping the 2D Ising Model into game theory, temperature is analogous to troop cohesion. Perfect cohesion within a domain assures only one side controls it (i.e., all its lattice sites' spins are aligned). Suboptimal coordination introduces imperfections, which allows, for instance, red to seize a few isolated tactical objectives in an otherwise blue domain. Troop cohesion is a function of both troop morale and the usage of effective communications. If communications equipment is unreliable or if troop morale is dangerously low, then soldiers and sergeants fail to comply with orders handed down from majors and generals (i.e., the strategic level), and army cohesion begins to disintegrate. A higher temperature in the physics version of the Ising model corresponds to a lower efficiency in communications and/or lower morale in the economics counterpart, either way decreasing pan-lattice cohesion. Imperfections (due to thermal fluctuations in physics, due to the fog of war and officer errors in game theory) first arise *within* the given domains, meaning that the major is the first level to experience difficulty navigating the decision space when his subordinates begin to falter. Although riddled with imperfections, the domains are still discernable at this point (i.e., Figure 2), which allows the general to still direct troops to any given domain. At the critical temperature, troop compliance completely disintegrates. Beyond the critical temperature (i.e., the point of no return in degrading pan-lattice communications/morale), strategic planning shatters altogether and the lattice spin sites (i.e., control of tactical objectives) is completely determined *locally* by sergeants and soldiers (who are failing to receive orders) rather than their commanding officers. They still fight for the time being, affecting neighboring spins, but pan-lattice cohesion (i.e., spin correlation) evaporates. As confusion arises in blue's ranks, red becomes equally perplexed. Red sergeants and majors partially base their strategies upon their blue counterparts' choices (or at least the belief of blue's choices), so chaos in blue's decisions translates to a greater degree of chaos in red's decisions. At this point, the lattice's magnetic susceptibility is at its maximum, providing ample opportunity for an external event to orient virtually all spins in a single direction. On the battlefield, this corresponds to a massive propaganda victory which shreds morale (such as enemy usage of nuclear weapons) or an electronic warfare assault (e.g., EMP, cyberattack) which cripples communications. Such a drastic external event is tantamount to an external magnetic field being applied to the lattice in physics. The Appendix includes a Python program which aptly simulates this arrangement.

Soldier (Simultaneous Tactical Game)

In this rendering, soldiers are quite simplified. Their sole mission and objective is "shoot before being shot" – a simultaneous game between them and the adversary's soldiers. Soldiers cannot defect, go AWOL, or abandon their compatriots. In the below utility function (equation 1), soldiers individually make choices to maximize their survivability. The variable R determines how cautiously a soldier chooses to play his role, prioritizing his survivability (or not). Perhaps he is aggressive and takes a big risk (i.e., standing bolt upright in the midst of oncoming fire to get a better shot), but provides a massive tactical payoff. Perhaps he is cautious (i.e., hunkering down in his foxhole), that ensures survival but impairs his impact. A higher R value means that the soldier is playing his game more cautiously (within the constraints of the mission assigned by his sergeant, which obviously ranges in danger), but is *generally* less impactful in the moment. Additionally, θ dictates how closely the soldier follows his sergeant's orders. A higher value of θ corresponds to a higher adherence to orders and therefore a higher probability that the soldier survives, given that he has the support of his compatriots (if they are also cohesively following orders). Both R and θ range from zero to 1, with maximum survivability entailing both R and θ equal 1. A soldier balances R and θ to maximize his survivability utility function, s_b. As the next section details, the number of surviving blue and red soldiers (s_b and s_r, respectively) on a given lattice site partially determine who controls it.

$$s_b = R\theta \tag{1}$$

Sergeant (Sequential Tactical Game)

When a sergeant is tasked with seizing or defending a tactical objective (aka arrow on the 2D Ising lattice), he plays a sequential game to determine his approach. If the enemy is already entrenched at the site, the sergeant – with a fair degree of certainty – assesses the situation himself and acts accordingly. After all, most all of the variables that affect his decision of tactics are within eyesight or earshot. He physically *observes* the battlefield reality and proceeds accordingly. Each move, each choice in tactics (evidenced by movement of soldiers) one sergeant makes is *directly observed* by the other, who adjusts tactics accordingly, making this a sequential game. The Sergeant's moves vary depending upon his nation's tactical playbook, but typically would include: entrench, tactical retreat, frontal assault, delaying action, etc., selecting the tactics employed by the 10 or so soldiers at his lattice site. During lulls in combat, the sergeants radio updates back to their commanding major, periodically providing the major with an updated image of the battlefield in his domain. Since a sergeant's entire game is to secure his assigned tactical objective, his utility function is to maximize friendly control of the lattice site. For our purposes, control is equivalent to μ_{site} (see Equation 2), which is the sergeant's utility function. For a given lattice site (aka tactical objective), Equation 2 governs which side (red or blue) wields control of it. In Equation 2, s_b is the number of blue troops, ν_b is a quality factor for blue troops, ranging from 0 to 1, and τ_b is a weighing factor for the tactics the blue sergeant elects to employ. ν_b increases with better training, equipment, and physical fitness/rest for blue's troops. Similarly, τ_b quantifies the expected effectiveness of tactics, depending upon the sergeant's tactical playbook. Paralleling blue, red's quantity of troops is s_r with a quality factor of ν_r, employing tactics with predicted impact τ_r. Notably, the sergeant makes the most frequent decisions (i.e., hundreds per day) to adjust his choice of tactics (and therefore expected τ value), but they are of lesser impact than those of his superior officers. In other words, the frequency of decision-making is inversely proportional to the decisions' importance.

$$\mu_{site} = \tau_b \nu_b s_b - \tau_r \nu_r s_r \tag{2}$$

Equation 2 demonstrates that we can tweak the otherwise chaotic nature of spin alignment to favor one direction (i.e., spin up) over the other, depending upon who wields the greater control at this lattice site. Obviously, a site swarming with well-trained red troops employing effective tactics pitted against a measly bunch of blue stragglers heavily favors red control of the site, reflected in the site's spin being spin down (aka red) with sizable magnitude. We plug equation 2 into the 2D Ising Model Hamiltonian (equation 3; a physics expression of energy for the system) and computationally solve for the time-evolution of the war, such as in the Appendix's sample Python program. The first term of equation 3 describes the coupling interaction between neighboring lattice sites (i.e., troops' impact from one tactical objective upon a neighboring tactical objective). The second term accounts for the impact of an external magnetic field (in our case external, universal events like EMPs or nuclear weapons which affect the entire war's progression simultaneously) upon the site's spin. Computationally, the Hamiltonian is calculated, then utilized to crafts the visuals (i.e., Figures 1-4).

$$H = -J \sum_{\langle i,j \rangle} \mu_{site_i} \mu_{site_j} \; - h \sum_{i} \mu_{site_i} \tag{3}$$

Major (Simultaneous Strategic Game)

Continuing up the chain of command, we again encounter a simultaneous game – played by the major. Attempting to grapple with the fog of war and aggregate periodic reports radioed in by his subordinate sergeants, a major is in charge of distributing troops across a given domain. The major determines precisely how many sergeants (and soldiers) are to be deployed to each lattice site within his domain, deciding based upon his importance weighting term (sigma), which dictates how the finite quantity of troops under his command should be arranged across his domain. His moves are simple: add, subtract, or keep constant the quantity of troops at the lattice sites in his domain. Since his entire picture is constrained to advancing the front line across his domain, his utility function (Equation 4) is maximizing friendly control over this domain. Since he cannot *directly observe* current realities across the entire lattice (it stretches beyond his line of sight, making him dependent upon the periodic radio updates from his sergeants), the major is operating with time-delayed, imperfect information from his sergeants (updating him on the status of various lattice sites, which he then uses to gauge the enemy's intent in each site based on *beliefs*), making his game simultaneous as the frontline ebbs and flows. He cannot know with certainty the situation at any given lattice site since he is not physically present there. Without observability of his adversary's moves and basing his actions on beliefs of the enemy's intentions, the Major is engaging in a simultaneous Bayesian game. Translating strategic goals into battlefield actions, the major integrates the general's orders with aggregated information from his subordinate sergeants, setting lattice sites' importance at an intermediate timestep (larger than the sergeant's, smaller than the general's). The major increases/decreases a given lattice site's sigma factor, adjusting its priority as the battle dictates. An increase in importance necessitates a reconfiguration of existing deployments across the domain, redistributing troops to this site.

In the next section, troop movements across the lattice are treated in detail. There is a finite constant quantity of troops (and materiel) in any given domain for a major to allocate as he sees fit. This quantity is determined by the general and redetermined at each general's timestep. Conveniently, the 2D Ising Model only considers nearest-neighbor interactions, meaning that troops at one tactical objective can only influence troops on neighboring tactical objectives. If the major intends on defortifying a lattice site (i.e., *decreasing* its importance weighting, thereby reallocating troops elsewhere), any resources (troops and materiel) stationed there are moved to another lattice site. In reality, this takes time to move people and objects, which sees these resources "disappear" from the lattice for a brief transition time, then "reappear" in the new lattice site at which they are stationed.

$$\mu_{domain} = \sum_{i=1}^{n} \sigma_i \, \mu_{site_i} \tag{4}$$

General (Sequential Strategic Game)

Upon arriving at the general's tier of the game, we again revert to a sequential game. Since the strategic sphere progresses sufficiently slowly from a general's perspective (i.e., moves are clearly known by either side since they manifest over the course of weeks/months and provide the other side time to react accordingly; ordering the sequence of moves), it diverges from the major's simultaneous game and is treated as a sequential game. Realistically, generals issue orders to subordinates (i.e., colonels), who then coordinate majors, but since these subordinates' games are straightforward subgames of the general's game (i.e., a broad-sweeping sequential game), it is condensed down (as are many other ranks at other game levels) into the general's game for brevity. Generals dictate the importance (gamma; an integer) of securing the various domains in their utility function (equation 5), limiting their moves to: add, subtract, or keep constant the quantity of troops in a given domain. Some domains are inherently more valuable than others (i.e., a capital city is more desirable than surrounding farmland) and some values vary with time. A domain with an airport, for instance, gains importance if the general is planning to land reinforcements for a nearby assault. The domain containing this airport can be shrunk to accommodate merely the airport (perhaps with various runways/buildings comprising a handful of tactical objectives on the lattice) plus the directly neighboring locales (i.e., highway of entrance, nearby hilltops, etc.). The general integrates these considerations with the information passed up from his subordinate majors, then formulates an importance weighting based upon a confluence of his training, experiences, personality, and beliefs about the enemy's plans and intentions.

As the strategic picture chaotically evolves, either from intrinsically unpredictable tactical outcomes or unforeseen strategic obstacles, there exist dual feedback loops. Tactical realities passed up from subordinates, paired with reshuffling strategic priorities (at the major's or general's scale) dictate the general's gamma values, which then percolate down the ranks, affecting troop deployments down to the sergeant's level. Over the course of the war, however, importance weightings shift – often unpredictably. If blue forces begin to run out of fuel, for instance, their new strategic priority will be oil-rich land (i.e., Germany's Operation Barbarossa in World War 2), which before may have been deemed an unnecessary risk to secure. Such dynamics are aptly captured by the Ising Model's functionality.

$$\mu_{lattice} = \sum_{j=1}^{m} \gamma_j \, \mu_{domain_j} \tag{5}$$

Modeling Troop Flow Across the Lattice

Given the 2D Ising Model, we employ the below set of ordinary differential equations to model troop flow throughout lattice domains from site to neighboring site and put it in terms of our established Ising Model variables. Conveniently, the 2D Ising Model only considers nearest-neighbor interactions, meaning that troops at one tactical objective only influence troops at neighboring tactical objectives. This corresponds neatly with real-world troop dynamics, as only nearby forces arrive in time or (from a distance) provide supporting fire to affect the outcome of a given tactical objective's conflict. Refer to Figure 5 to visualize the setup, which is a modified SIR Model from epidemiology (Prodanov, 2022; Rapatski, 2021). Note that the flow of troops across the lattice is identical to the flow of ammunition, supplies, and other war materiel. While troop quantities (i.e., T_1, T_2, T_3, and T_4) should be expressed as a decimal percentage of overall troops in the domain, each influx/recall of reinforcements from the general requires the model to be renormalized. Once additional troops enter/leave domain x, the decimal values of T_1 through T_4 are recomputed. Here, we model troop flow in terms of the Ising model variables, including the variables representing decisions within the games.

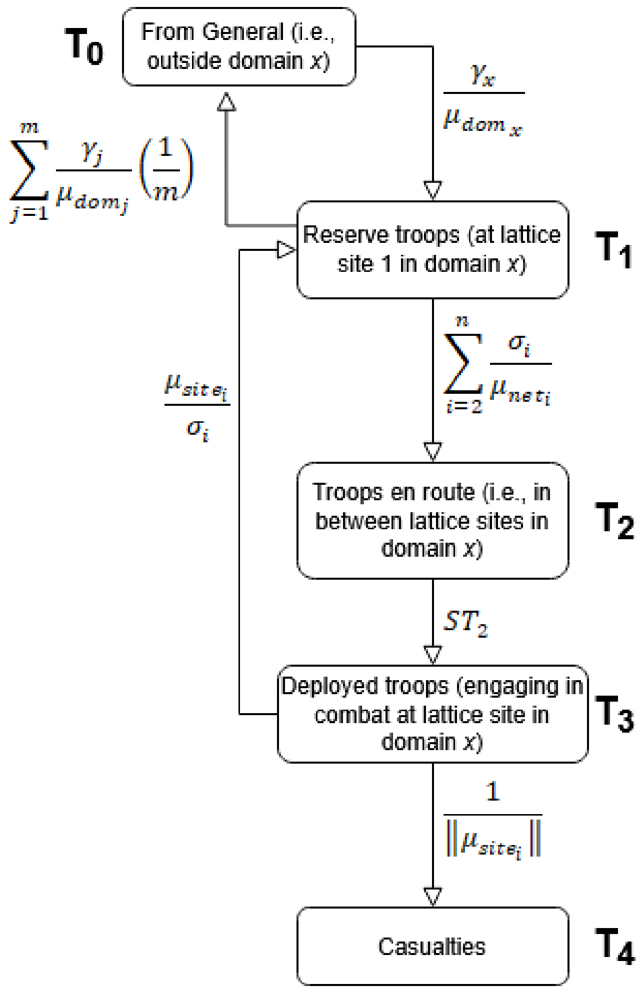

Figure 5: Modified SIR model showing troop flow across 2D Ising Lattice. Author's original work.

$$\dot{T}_0 = -\frac{\gamma_x}{\mu_{dom_x}} + \sum_{j=1}^{m}\frac{\gamma_j}{\mu_{dom_j}}\left(\frac{1}{m}\right)$$

$$\dot{T}_1 = \frac{\mu_{site_1}}{\sigma_1} - \sum_{i=2}^{n}\frac{\sigma_i}{\mu_{site_i}} + \frac{\gamma}{\mu_{dom}} - \sum_{j=1}^{m}\frac{\gamma_j}{\mu_{dom_j}}\left(\frac{1}{m}\right)$$

$$\dot{T}_2 = \sum_{i=2}^{n}\frac{\sigma_i}{\mu_{net_i}} - ST_2$$

$$\dot{T}_3 = ST_2 - \frac{\mu_{site_i}}{\sigma_i} - \frac{1}{\|\mu_{site_i}\|}$$

$$\dot{T}_4 = \frac{1}{\|\mu_{site_i}\|}$$

The general determines how to redistribute troops from domain to domain, carefully balancing control of each to conquer the entire lattice. The troops at T_0 are at the general's disposal, outside of domain x (i.e., the domain within which the populations of T_1, T_2, T_3, and T_4 are located). The general, based upon his importance weighting and the inverse of a domain's control, determines how many troops to dispatch to that domain. This dictates the first term in $\dot{T}_0$. When these troops arrive in a domain, they are added to a lattice site arbitrarily numbered 1 for the purposes of this discussion – a site which is relatively tranquil (i.e., uncontested) as these troops will have to accustom themselves to the local battlefield's status and orient themselves in the new domain. This site could be more generally termed a variable, but the "1" identifier reminds us that it is not included in the neighboring summations, which began at $i = 2$. If the major maintains excessive troops in his domain, the general recalls them and redistributes them to another domain as needed, providing us the second term in $\dot{T}_0$. m is the number of domains on the lattice. The $1/\ m$ shows that the general's importance weighing is divided over all the domains on the map, not just domain x. With a finite quantity of "importance" (i.e., γ) to distribute across the lattice's domains, the more domains there are, the less valuable any given domain is, on average.

For T_1, the first term represents the portion of troops at lattice site 1 which are considered "reserves" (i.e., above and beyond what that particular site requires to assert friendly control). For instance, a tactical objective with 10,000 friendly troops and only a few dozen enemy troops would classify the vast majority of friendly troops as "reserves" to be dispatched elsewhere, as needed. The larger the control and smaller the importance of a lattice site, the more troops at it will be in reserve. The first summation term encompasses the sum of other lattice sites within the major's domain, dictating that other sites that are of high importance and low control require a dispatchment of reinforcements (i.e., reserve units) from lattice site 1. n is the number of sites (i.e., arrows) within the domain. While the third term represents the reinforcements dispatched by the general to this major's domain, the fourth term accounts for excess reserve troops which the major returns to the general's stockpile.

If one wants to account for the fact that troops take time to move across the lattice, utilize T_2, otherwise ignore it, as its output feeds directly into T_3 without any contravening variables. Essentially, troops in transit between lattice sites "disappear" from the lattice, since they cannot be influential in any ongoing conflict at tactical objectives. T_2 accounts for those troops, with S as a constant of the troops' speed. If one desires to model army attrition from encountering guerilla resistance during transit, an additional arrow can be drawn from the T_2 box directly to T_4, with an additional term added to the expression for $\dot{T}_2$ to account for these resistive losses, analogous to Ohmic losses in electricity.

Once reaching their intended tactical objective (aka lattice site), the troops are considered “deployed” in the T_3 category. As time progresses, deployed troops either become casualties or, while victoriously securing a tactical objective, gradually transmute into reserve forces at that particular lattice site, freeing themselves for deployment elsewhere. Conversely, reserve troops stationed at a lattice site which suddenly becomes contested revert to deployed troops by reversing the sign (and flow) of the second term in $\dot{T}_3$. Note that $\dot{T}_4$, the casualty rate of friendly troops, is inversely proportional to the magnitude of friendly control at a given lattice site. Intuitively, this makes sense. If friendly forces are rapidly losing troops at a lattice site, they are failing to assertively control that tactical objective. Inversely, a slow casualty rate corresponds to tight control over a tactical objective.

Conclusion

While the 2D Ising Model certainly does not holistically encapsulate the complexities of war, it provides a method to readily visualize the levels of interrelated games, their corresponding utility functions, and their interdependence. Since the 2D Ising Model is a well-established computational framework within condensed matter physics (see Appendix for Python program which simulates war using the 2D Ising code), this mapping also provides conflict analysts a robust simulation environment to experiment with, applying it to visualize and evaluate various hierarchies of games. Since the dual feedback loops (control from the sergeant’s level migrating upwards and informing troop movement by major/general and the general/major’s importance weighting dictating troop deployment back down to sergeant level) are built into the modeling, real-world complexity is preserved which would be otherwise overlooked in simplified approaches, rendering a more accurate model. The visualization power of the Ising Model predominates, allowing analysts to employ a highly intuitive vehicle to showcase war realities. Further evaluations of how to optimally interconnect hierarchically nestled games is advised within game theory and economics, as is formulating potential business applications from this framework.

Acknowledgements

While the author maintains an interest in game theory economics and is currently preparing to start a PhD in theoretical plasma physics, the origin of this idea is much less academic. Attempting to convince his friend to try his favorite videogame, “Planetside 2”, the author realized he had to analyze why the game was so complex – and that there was something that emerged from the complexly layered games within Planetside 2 that distinguished it from other first-person shooter games.

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Appendix

See attached file for Python code.

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