Imperfect Competition in Online Auctions*

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Abstract

We study online auctions, where two sellers sequentially choose reserve prices and then hold ascending auctions. Buyers are able to bid in both auctions and can switch between them as frequently as they like. In contrast to competition in traditional auctions, where sellers simultaneously choose reserve prices and each buyer commits to participating in a single auction, in which an equilibrium exists only in mixed strategies, we show that the sequential online auction game has a pure strategy equilibrium. This equilibrium is inefficient because both sellers choose a reserve price higher than the marginal costs (but still smaller than a collusive outcome).

Keywords: Internet Auctions; Competing Auctions; Incentive-Compatible Mechanisms; Inefficiency

JEL Codes: C73, D44

1 Introduction

E-commerce has substantially transformed ordinary retail markets. It has also influenced the evolution of selling mechanisms and their contextual applications.

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Sealed-bid auctions were prevalent before the advent of the Internet, but have lost their popularity due to a drastic improvement in the communication technologies and reduction of search costs for buyers.

One of the important attributes of e-commerce is the ease of trading. Previously, companies had to incur fixed costs to set up at least one distribution channel. Reselling those items after the purchase was also problematic due to high search and coordination costs. These days, any individual may almost costlessly bring a product to an online consumer-to-consumer (C-2-C) market, whether for the purposes of resale or as a uniquely crafted item. The latter tendency produces a distinctive environment in which there may be only a few sellers offering small number of homogeneous goods to a large pool of buyers.

C-2-C platforms vary not only in their purpose, but also in the selling mechanisms that are available to sellers. The most popular selling mechanisms are posted prices, auctions and auctions with a buy-it-now price (eBay also has the "best offer," which is effectively a first-price sealed-bid auction). There are several broad online markets where auctions are dominating as selling mechanisms, e.g., eBid, AuctionZip or LiveAuctioneers. Other platforms are more targeted. For example, Propertyroom helps law enforcement agencies to auction off seized, stolen, abandoned and surplus items, which is required by law. Shopgoodwill receives all of its goods (mostly clothing and household items) from donations and then auctions them off to the public. It further uses resulting profits to provide job training in the USA. Listia does not use money at all, but gives credits to consumers for donating items, which later could be used to bid at the website.

The largest online C-2-C market is hosted by eBay, but because sellers can list their items using several mechanisms on this platform, in addition to more complicated behavior by buyers that are faced with the same objects being sold using different mechanisms, the choice of the mechanism itself may be endogenous. Hence, the analysis in this note is focused on auction-only online markets like the platforms described above.

The sparse theoretical and empirical literature on competing online auctions investigates the behavior of buyers or final purchase prices. We, on the other hand, are mainly interested in how competing sellers choose their reserve prices. We show that even with the same marginal costs, the first-arriving seller chooses a reserve price just low enough to safeguard against being undercut by the second seller, and in equilibrium the first-arriving seller makes more profit than the second-arriving seller. In addition to characterizing an equilibrium in the above environment, this note has a methodological contribution. We show how the approach of Myerson (1981) can be extended to a case with two sellers receiving some split of the expected revenue generated from the buyers.

Peters and Severinov (2006) prove that when there are many sellers and buyers in online-auction markets, the reserve prices set by the sellers are equal to their marginal costs. In contrast to sealed-bid auctions characterized by simultaneous choice of reserve prices, it is unlikely that in online markets sellers choose reserve prices simultaneously. Rather, a seller who comes to the market first, chooses a reserve price expecting a subsequent arrival of another seller. In principle, sellers may have a good estimate of how many competitors to anticipate. Such a strategic environment may be framed as a Stackelberg-like model where sellers choose reserve prices, and our results are consistent with the standard symmetric Stackelberg model, in which the first-moving seller has an advantage and earns a higher profit.

Our note is related to Burguet and Sákovics (1999), who show that the results of McAfee (1993) and Peters and Severinov (1997) hold only for large markets where many sellers offer sealed-bid auctions. The crucial feature of the environment considered by this literature is the commitment of buyers, who could no longer switch to another auction after placing a bid in one of them. Burguet and Sákovics (1999) argue that in a duopoly the reserve prices are no longer driven to marginal costs. The authors consider simultaneous choice of reserve prices by the sellers and find that the equilibrium exists only in mixed strategies. When the choice of reserve prices is sequential (which reflects the observed regularities of online markets), we show that there is a unique equilibrium outcome. Due to differences in the behavior of buyers faced with either sealed-bid or ascending auctions, Burguet and Sákovics (1999) could not use the marginal revenue approach (Myerson (1981), Bulow and Roberts (1989), Bulow and Klemperer (1994)), which is applicable in our analysis and allows us to tremendously simplify calculations further generalize our results to any selling mechanisms in which only the highest valued buyers are awarded units.

We show that just like in the environment considered by Burguet and Sákovics (1999), competition between two sellers competing in online auctions is not enough to drive reserve prices to marginal costs. To our knowledge, there is no empirical literature examining the structure of reserve prices in online auction markets. Our theory predicts variation to exist even with two sellers. This contrasts with a monopolist who sells items by auctions at the same optimal reserve price and a competitive market in which reserve prices are equal to marginal costs. The monopolist outcome may also arise if competing duopolists were to collude. Hence, the absence of variation in the reserve prices on particular segments of C-2-C markets could potentially be used as a test for collusion.

In the next section we describe the model. In section 3 we describe the sellers' profits directly and then adapt the revenue equivalence theorem to rewrite the sellers' profits. In section 4 we describe the equilibrium. Section 5 provides an example with three buyers with uniformly distributed values and shows how the reserve price of the first-moving seller is just high enough to discourage the second-arriving seller from undercutting. We conclude in section 6 by considering online auctions (in which buyers can bid simultaneously in both auctions), but in which the sellers choose reserve prices simultaneously to better understand the role of sequentially chosen reserve prices.

2 The model

There are two sellers with identical costs (normalized to zero) — a and b — each possessing a single unit. There are n buyers, each demanding a single unit. The values of the buyers are i.i.d, drawn from distribution $F(\cdot)$ with support $[0, \overline{v}]$; $F(\cdot)$ is differentiable with everywhere positive density $f(\cdot)$. Let the vector of valuations be $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ and the vector of sorted values be $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ with $x_1 \geq x_2 \geq \ldots \geq x_n$. In other words, the elements of $\mathbf{x}$ are order statistics. Let $f_k(x)$ denote the marginal density function of the k^{th} highest order statistic and let $f_{1:k}^{(n)}(x_1, x_2, \ldots, x_k)$ denote the joint density of the k^{th} highest order statistics. All players are risk-neutral. A seller's ex post profit is equal to the payment he receives. A buyer's ex post payoff equals his value if he wins a unit, less any payment he makes.

Our model covers one story exactly and another story approximately. The exact story has a centralized auction taking place at some specified date. Prior to this date, seller a arrives and selects reserve price r_a and then seller b arrives, observes r_a, and selects reserve price r_b. Each buyer submits a sealed bid. The allocation is according to the seller-offer double auction, which works as follows. Make a single list, sorting the reserve prices and bids from highest to lowest, with ties ordered randomly. Set price P equal to the reserve price or bid in the n^{th} lowest position on this list. All sellers amongst the n^{th} lowest positions will sell a unit and receive P dollars; all buyers with values in the remaining 2 "highest" positions will purchase a unit and pay P dollars. The remaining sellers and buyers do not transact. This means that the price paid by each winning buyer is set by either a losing buyer or a seller, but not his own bid. Thus, every buyer has a dominant strategy to bid his value so that he wins a unit if and only if profitable to do so. But a seller can both sell a unit and set the price, distorting his incentive to set a reserve at his cost (here zero).

The approximate story is that the selling procedure is decentralized. Sellers arrive sequentially and set reserve prices as before, but this time each seller activates a separate ascending price auction. Once all of the reserve prices are chosen, the auctions begin and buyers can bid in any of the auctions. The auctions end when some period of time passes with no further bids. Peters and Severinov (2006) have treated a similar environment in which sellers choose reserve prices simultaneously and have independent private costs. They further assumed a finite grid of allowable prices in the auctions coinciding with the supports that the sellers and buyers draw their costs and values from. A key result in Peters and Severinov (2006) is that there exists a perfect Bayesian equilibrium in the bidding game, in which each buyer bids minimally as needed, only bidding in the auction with the lowest price whenever that buyer is not already the highest bidder in one of the auctions and that the lowest price is less than his value. Such a strategy could be implemented by a computerized algorithm or machine proxy. If all buyers used it and if the bid increments became finer and finer, the selling procedure would be strategically and outcome equivalent to the seller-offer double auction described above, in the same way that Vickrey (1961) found strategic equivalence between a single-unit second-price auction and an ascending price auction in a private-values setting.

3 Sellers' profits

In the seller-offer double auction or in decentralized ascending price auctions, only the buyers with the highest valuations win units, as described in the prior section. We next give seller profit functions based on whether the seller has the lower or higher reserve price. Name the reserve prices such that $r_2 \geq r_1$. We first treat the case when $r_2 > r_1$. The seller with reserve price r_2 sells a unit at price r_2 if $x_1 \geq x_2 \geq r_2 > x_3$ and at price x_3 if $x_1 \geq x_2 \geq x_3 \geq r_2$ for expected profit of:

$$\begin{aligned}\pi_2(r_2) = &\int_{x_3=0}^{x_3=r_2} \int_{x_2=r_2}^{x_2=\overline{v}} \int_{x_1=x_2}^{x_1=\overline{v}} r_2 f_{1:3}^{(n)}(x_1, x_2, x_3) dx_1 dx_2 dx_3 + \\ &\int_{x_3=r_2}^{x_3=\overline{v}} \int_{x_2=x_3}^{x_2=\overline{v}} \int_{x_1=x_2}^{x_1=\overline{v}} x_3 f_{1:3}^{(n)}(x_1, x_2, x_3) dx_1 dx_2 dx_3.\end{aligned} \tag{1}$$

The seller with reserve price r_1 sells a unit at price r_2 if $x_1 \geq x_2 \geq r_2 > x_3$ and at price x_3 if $x_1 \geq x_2 \geq x_3 \geq r_2$ as before, and also at price r_1 if $x_1 \geq r_1 > x_2$ and at price x_2 if $r_2 \geq x_2 \geq r_1$ for the expected profit of:

$$\begin{aligned}\pi_1(r_1, r_2) = \pi_2(r_2) + &\int_{x_2=0}^{x_2=r_1} \int_{x_1=r_1}^{x_1=\overline{v}} r_1 f_{1:2}^{(n)}(x_1, x_2) dx_1 dx_2 + \\ &\int_{x_2=r_1}^{x_2=r_2} \int_{x_1=x_2}^{x_1=\overline{v}} x_2 f_{1:2}^{(n)}(x_1, x_2) dx_1 dx_2.\end{aligned} \tag{2}$$

Despite the particulars of the payments, the revenue equivalence theorem (Myerson (1981), Riley and Samuelson (1981), Krishna (2009)) indicates that what matters is the allocation of units: in a single-unit demand independent private values setting (as in our model), in any incentive compatible mechanism in which a buyer with value 0 gets an expected payoff of 0, the expected revenue equals the expected marginal revenue of the buyers awarded units, where marginal revenue is defined as $MR(z) \coloneqq z - \frac{1-F(z)}{f(z)}$. We may thus express the profit functions as summarized in the following proposition.

Proposition 1. *An equivalent way to express the profit functions is:*

$$\pi_2(r_2) = \frac{1}{2} \int_{x_2=r_2}^{x_2=\overline{v}} \int_{x_1=x_2}^{x_1=\overline{v}} \Big(MR(x_1) + MR(x_2) \Big) f_{1:2}^{(n)}(x_1, x_2) dx_1 dx_2$$

and

$$\pi_1(r_1, r_2) = \pi_2(r_2) + \int_{x_2=0}^{x_2=r_1} \int_{x_1=r_1}^{x_1=\overline{v}} MR(x_1) f_{1:2}^{(n)}(x_1, x_2) dx_1 dx_2 + \int_{x_2=r_1}^{x_2=r_2} \int_{x_1=x_2}^{x_1=\overline{v}} MR(x_1) f_{1:2}^{(n)}(x_1, x_2) dx_1 dx_2.$$

Proof. In the seller-offer double auction, buyers have a dominant strategy to bid their values, so that a unit is sold to each of the two highest order statistics whenever $x_2 \geq r_2$ and a unit is sold to only the highest order statistic when $x_1 \geq r_1 > x_2$ or $r_2 > x_2 \geq r_1$. The aforementioned revenue equivalence theorem applies and gives $\pi_1(r_1, r_2) + \pi_2(r_2)$ as specified in this proposition for the aggregate expected revenue generated from the buyers. The split of revenue as designated also results from the revenue equivalence theorem. To see this, construct as follows a dominant strategy direct revelation game in which buyers report their values that ex post replicates the payment received in the seller-offer double auction by the seller with the higher reserve price. Each buyer is awarded a $1/2$ unit if and only if his report exceeds a price determined by the reports of his opponents and r_2. He also pays $1/2$ of this price whenever so awarded. This price is infinity if the maximum of the other buyers' reports is below r_2; is r_2 if the maximum of the other reports is above r_2, but the remaining reports are below r_2; is the second highest of the other reports when this value exceeds r_2. □

For the case when $r_2 = r_1 \equiv r$, each seller gets expected profit

$$\pi_0(r) = \frac{1}{2}\pi_1(r, r) + \frac{1}{2}\pi_2(r).$$

since the order of their reserve prices in the list used in the seller-offer double auction is random (or in the ascending auctions if only one buyer has a value exceeding this reserve price, he would randomly choose which auction to place his bid).

We maintain the following standard assumption throughout our paper (see Myerson (1981)).

Assumption 1. *The marginal revenue function* $MR(z) := z - \frac{1-F(z)}{f(z)}$ *is regular: that is, it is continuous and strictly increasing.*

We will also make use of the following quick result introduced in Bulow and Roberts (1989), which can be shown using integration by parts:

Lemma 1. *For all* p *with* $0 \le p \le \overline{v}$*, we have:*

$$\int_p^{\overline{v}} MR(z)f(z)dz = p[1 - F(p)].$$

The next three lemmas establish important properties of the profit functions.

Lemma 2. *For all* (r_1, r_2) *with* $0 < r_1 \le r_2 < \overline{v}$*, we have:*

$$\pi_1(r_1, r_2) > \pi_2(r_2).$$

Proof. This follows immediately from equations (1) and (2). □

Lemma 3. *For all* r *with* $0 < r < \overline{v}$*, we have:*

$$\pi_1(r, r) > \pi_0(r).$$

Proof. By Lemma 2, $\pi_1(r, r) > \pi_2(r)$. By definition, π_0 is a convex combination of $\pi_1(r, r)$ and $\pi_2(r)$ and thus lies somewhere in between: $\pi_1(r, r) > \pi_0(r) > \pi_2(r)$. □

Lemma 4. *The function* $\pi_2(\cdot)$ *defined on* $[0, \overline{v}]$ *is single-peaked, and reaches its peak at* $r_2^* := \psi^{-1}(0)$*, where* $\psi(r_2) := r_2 + MR(r_2)$*. Each function in the family* $\{\pi_1(\cdot, r_2)\}_{r_2 \in [0, \overline{v}]}$*, with* $\pi_1(\cdot, r_2)$ *defined on* $[0, r_2]$*, is single-peaked and reaches its peak at* $min\{r_1^*, r_2\}$*, where* $r_1^* := MR^{-1}(0)$*.*

Proof. Use Proposition 1 to get:

$$\begin{aligned}
\frac{d\pi_2(r_2)}{dr_2} &= -\frac{1}{2}\int_{x_1=r_2}^{x_1=\overline{v}} \Big(MR(x_1) + MR(r_2)\Big) n(n-1)f(x_1)f(r_2)F^{n-2}(r_2)dx_1 \\
&= -\frac{1}{2}n(n-1)f(r_2)F^{n-2}(r_2)\Big[(1 - F(r_2))r_2 + MR(r_2)(1 - F(r_2))\Big] \\
&= -\frac{1}{2}\underbrace{n(n-1)f(r_2)F^{n-2}(r_2)(1 - F(r_2))}_{f_2(r_2)\,\ge\,0}(r_2 + MR(r_2))
\end{aligned}$$

where Lemma 1 gives the second equality. This derivative equals 0 when r_2 is 0 or $\overline{v}$, but otherwise takes sign opposite of $\psi(r_2) := r_2 + MR(r_2)$. Because $\psi(0) < 0 < \psi(\overline{v})$ and $\psi(r_2)$ is continuous and strictly increasing by Assumption 1 (regularity), there is a unique value of r_2 in the interior of $[0, \overline{v}]$ with $\psi(r_2) = 0$: $r_2^* = \psi^{-1}(0)$. Thus, in the interior of $[0, \overline{v}]$, $d\pi_2(r_2)/dr_2$ begins positive, equals zero at r_2^*, and turns negative, thereby giving the single-peakedness of $\pi_2(r_2)$.

Next, use Proposition 1 to get:

$$\begin{aligned}\frac{\partial \pi_1(r_1, r_2)}{\partial r_1} &= -\int_{x_2=0}^{x_2=r_1} MR(r_1)n(n-1)f(r_1)f(x_2)F^{n-2}(x_2)dx_2 \\ &= -MR(r_1)nf(r_1)\int_{x_2=0}^{x_2=r_1} (n-1)f(x_2)F^{n-2}(x_2)dx_2 \\ &= -\underbrace{nf(r_1)F^{n-1}(r_1)}_{f_1(r_1)\,\geq\, 0} MR(r_1).\end{aligned}$$

A similar argument to the prior paragraph gives the single-peakedness of $\pi_1(r_1, r_2)$ at $r_1^* = MR^{-1}(0)$ whenever $r_1^* \leq r_2$ and otherwise at r_2, noting that $\pi_1(\cdot, r_2)$ is only defined on $[0, r_2]$. □

Lemma 5. *The following ranking holds:* $0 < r_2^* < r_1^*$.

Proof. The function $\psi(r_2) = r_2 + MR(r_2)$ is strictly increasing and continuous by Assumption 1. By definition, $MR(r_1^*) = 0$ and $\psi(r_2^*) = 0$. The result follows from $\psi(0) = MR(0) = -1/f(0) < 0$ and $\psi(r_1^*) = r_1^* + MR(r_1^*) = r_1^* > 0$. □

4 Equilibrium

With the profit functions of the sellers thus defined, we are now ready to investigate the equilibria. We consider only the sequential game between the sellers: buyers are presumed to behave as already described. The game begins with the null history. A pure strategy for seller a is to choose a reserve price r_a from $[0, \overline{v}]$. A mixed strategy is a probability distribution on $[0, \overline{v}]$. Each r_a from $[0, \overline{v}]$ gives rise to a different history. Seller b's pure strategy is a function $r_b(r_a)$ which states a price r_b in $[0, \overline{v}]$ upon observing seller a's reserve price r_a. A mixed strategy involves selecting a probability distribution over $[0, \overline{v}]$ for each r_a in $[0, \overline{v}]$. We only emphasize histories here to more compactly define our equilibrium concept. A subgame perfect equilibrium gives a strategy for each player, such that after every history, the payoff to a player whose move it is cannot be improved by this player unilaterally deviating to another strategy.

Consider the value $\overline{r}_1$ such that $\pi_1(\overline{r}_1, \overline{r}_1) = \pi_2(r_2^*)$. Note that $\pi_1(r_2^*, r_2^*) > \pi_2(r_2^*)$ by Lemma 2 and $\pi_1(0,0) = \pi_2(0) < \pi_2(r_2^*)$ by Lemma 4. Note also that $\pi_1(r,r)$ is strictly increasing in r for all $r \in [0, r_2^*]$. This follows because for $0 \leq r < s \leq r_2^*$, we have $\pi_1(r,r) < \pi_1(r,s) < \pi_1(s,s)$, where the first inequality comes from equation (2), and the fact that $\pi_2(r) < \pi_2(s)$ comes from the single-peakedness result of Lemma 4, which also justifies the last inequality. The intermediate value theorem then implies that $0 < \overline{r}_1 < r_2^*$.

Proposition 2. No subgame perfect equilibrium exists.

Proof. Consider the subgame that begins after seller a chooses some price $r_a \in (\overline{r}_1, r_2^*)$. If seller b prices higher than r_a, then he does best with r_2^* by Lemma 4. For $\varepsilon > 0$ small enough, pricing $r_a - \varepsilon$ is even better for the second-moving seller because $\pi_1(r_a - \varepsilon, r_a) \approx \pi_1(r_a, r_a) > \pi_1(\overline{r}_1, \overline{r}_1) = \pi_2(r_2^*)$, recalling from the previous paragraph that $\pi_1(r,r)$ is strictly increasing in this region. Pricing exactly r_a is worse for the second-moving seller than pricing $r_a - \varepsilon$ for small enough $\varepsilon > 0$ by Lemma 3. Because $\pi_1(r_a - \varepsilon, r_a)$ is increasing as $\varepsilon > 0$ goes to zero (by Lemma 4) and because there is no smallest $\varepsilon > 0$, no best response exists for the second-moving seller. □

No subgame perfect equilibrium exists because of the technicality shown in the above proof: the second-moving seller wants to price just below r_a but there is no price just below r_a. Thus, we adapt from Radner (1980) the ε-perfect equilibrium to our game: this equilibrium specifies a strategy for each player, such that after every history, the payoff to a player whose move it is cannot be improved by more than ε by this player unilaterally deviating to another strategy. That is, each player's strategy is an ε-best response to the strategies designated for the other players. For $\varepsilon = \infty$, any strategy profile forms an ε-perfect equilibrium, but taking ε sufficiently small rules out some strategy profiles as being equilibria.

Proposition 3. *There is a unique ε-perfect equilibrium outcome as ε goes to zero: the first seller prices $\overline{r}_1$ and the second seller prices r_2^*.*

Proof. The proof is by backward induction. Suppose seller a has chosen reserve price r_a. We examine the best response of seller b. If $r_a = \overline{v}$, seller b has a unique best response to price r_1^*, using Lemma 4 and noting that choosing reserve price $\overline{v}$ results in zero profit. If $r_a = 0$, observe that

$$\pi_0(0) = \frac{1}{2}\pi_1(0,0) + \frac{1}{2}\pi_2(0) = \pi_2(0) < \pi_2(r_2^*)$$

where the inequality is from Lemma 4. Thus, choosing reserve price r_2^* is better than matching with a reserve price of 0, and is therefore the unique best response. For the remaining cases of r_a, we may appeal to Lemma 3 to note that matching this reserve price is never a best response for seller b. From Lemma 4 it follows that seller b does best whenever he chooses a lower reserve price to get as close as possible to r_1^* and does best whenever he chooses a higher reserve price to get as close as possible to r_2^*.

Case 1: $r_1^* < r_a < \overline{v}$. Seller b can achieve $\pi_1(r_1^*, r_a)$ by pricing below r_a. Pricing above r_a yields less than $\pi_2(r_a)$. Because $\pi_1(r_1^*, r_a) > \pi_1(r_a, r_a) > \pi_2(r_a)$, choosing reserve r_1^* is seller b's unique best response.

Case 2: $r_2^* \leq r_a \leq r_1^*$. Seller b gets less than $\pi_2(r_a)$ by pricing above r_a and gets arbitrarily close to $\pi_1(r_a, r_a)$ by pricing just below r_a. Because $\pi_1(r_a, r_a) > \pi_2(r_a)$, pricing just below r_a is the unique ε-best response.

Case 3: $\overline{r}_1 < r_a < r_2^*$. From the proof to Proposition 2, pricing just below r_a is the unique ε-best response for seller b.

Case 4: $0 < r_a \leq \overline{r}_1$. Recall that $\overline{r}_1$ is defined such that $\pi_1(\overline{r}_1, \overline{r}_1) = \pi_2(r_2^*)$. For all $r_b < \overline{r}_1$ we have $\pi_1(r_b, r_a) \leq \pi_1(r_b, \overline{r}_1) < \pi_1(\overline{r}_1, \overline{r}_1) = \pi_2(r_2^*)$ where the strict inequality is by Lemma 4, the weak inequality comes from equation (2) and the fact that $\pi_2(r_a) < \pi_2(\overline{r}_1)$ is established by Lemma 4. Thus seller b has a unique best response to price r_2^*.

Armed with the best response of seller b, back up to the decision of seller a. If seller a sets any reserve price $r_a > \overline{r}_1$, then seller b will choose some lower price as described above. Thus, seller a will be the high-priced seller and earn at most $\pi_2(r_2^*)$. Alternatively, if seller a sets any reserve price $r_a \leq \overline{r}_1$, then seller b will choose price r_2^*. Hence, seller a will earn $\pi_1(r_a, r_2^*)$. By Lemma 4, he will get the most in this region by setting $r_a = \overline{r}_1$. By Lemma 2, $\pi_1(\overline{r}_1, r_2^*) > \pi_2(r_2^*)$, which produces the desired result. $\square$

Proposition 3 shows that reserve prices are not driven down to the sellers' marginal costs, resulting in inefficiency. In addition, seller a who moves first sets a lower price and earns a higher profit than seller b since $\pi_1(\overline{r}_1, r_2^*) > \pi_2(r_2^*)$ by Lemma 2. As a remark, it follows from the aforementioned revenue equivalence theorem that if sellers were to collude to maximize their joint profits, they would set both reserve prices at $MR^{-1}(0) = r_1^* > r_2^*$. Thus, in a non-cooperative game with sellers moving sequentially, the equilibrium results in more social surplus (including the buyers) than in a monopolized or cartelized market.

5 Numerical example

Suppose that there are $n = 3$ buyers, with values distributed (uniformly) on $[0, 1]$. Then, $F(v) = v$, $f(v) = 1$, and marginal revenue is $MR(z) = 2z - 1$. Using Proposition 1, the profit functions for sellers with the higher and lower reserve prices are:

$$\pi_2(r_2) = \frac{1}{2}\int_{x_2=r_2}^{x_2=1}\int_{x_1=x_2}^{x_1=1}(2x_1 - 1 + 2x_2 - 1)6x_2 dx_1 dx_2 = \frac{9}{4}r_2^4 - 4r_2^3 + \frac{3}{2}r_2^2 + \frac{1}{4}$$

and

$$\pi_1(r_1, r_2) = \pi_2(r_2) + \int_{x_2=0}^{x_2=r_1}\int_{x_1=r_1}^{x_1=1}(2x_1 - 1)6x_2 dx_1 dx_2 +$$
$$\int_{x_2=r_1}^{x_2=r_2}\int_{x_1=x_2}^{x_1=1}(2x_1 - 1)6x_2 dx_1 dx_2 = -\frac{3}{2}r_1^4 + r_1^3 + \frac{3}{4}r_2^4 - 2r_2^3 + \frac{3}{2}r_2^2 + \frac{1}{4}$$

noting that the joint density of the two highest order statistics is $f_{1:2}^{(3)} = 6x_2$.

We obtain $r_2^* = 1/3 \approx 0.333$ by solving $r_2 + MR(r_2) = 0$ or $r_2 + 2r_2 - 1 = 0$. This is the value of r_2 that maximizes $\pi_2(r_2)$ and can be alternatively found by solving $d\pi_2(r_2)/dr_2 = 0$. This gives $\pi_2(r_2^*) = 8/27 \approx 0.296$. In equilibrium, seller a prices $\overline{r}_1$ such that $\pi_1(\overline{r}_1, \overline{r}_1) = \pi_2(r_2^*)$, or $\overline{r}_1 \approx 0.190$. This makes seller b just prefer to price r_2^* rather than undercut $\overline{r}_1$ by ε. Thus, in equilibrium seller a obtains profit $\pi_1(\overline{r}_1, r_2^*) \approx 0.357$ and seller b obtains profit $\pi_2(r_2^*) \approx 0.296$. The profit functions and equilibrium prices are illustrated on Figure 1.

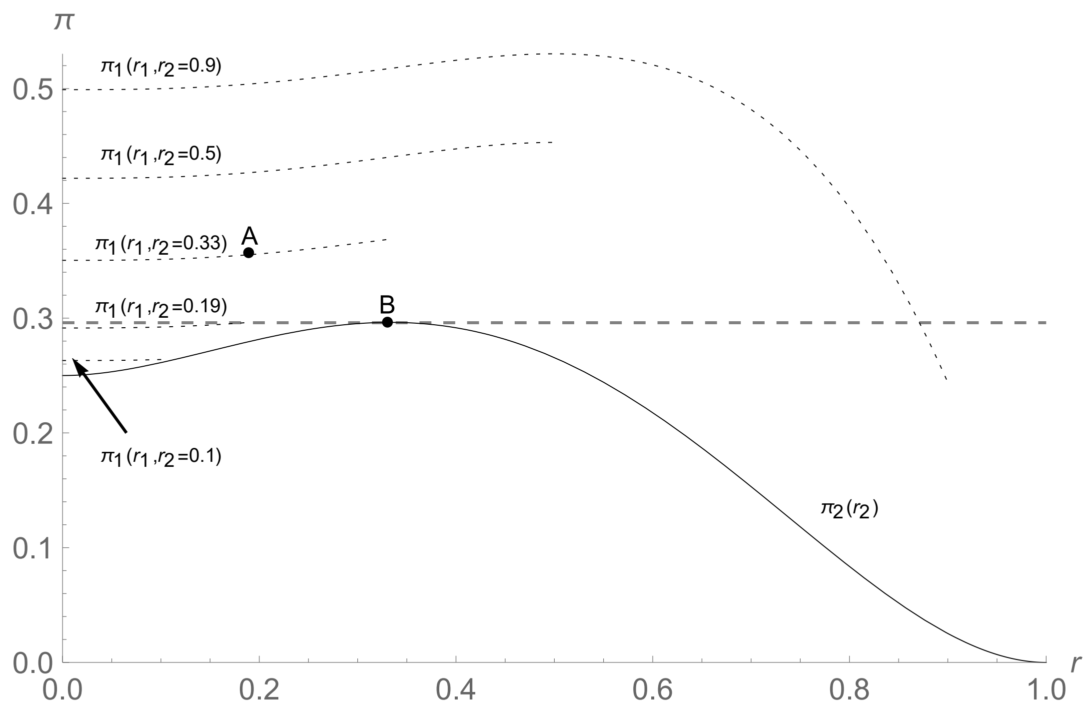
6 Conclusion

In this note we analyzed imperfect competition in online markets where two sellers enter the market sequentially and list their items by ascending auctions. We showed that the equilibrium outcome is unique with the first-arriving seller setting a low reserve price, and the second seller setting a higher reserve price. The first-moving seller receives larger expected profit, which is consistent with the first-mover advantage of the Stackelberg model. The equilibrium outcome is inefficient, because both reserve prices are set higher than the sellers' marginal costs.

Two more factors drive the results. The first one is the ability of buyers to switch costlessly between auctions, which is a likely feature of online markets. In the extreme case, buyers may procure bots scanning for desired goods across different digital auction platforms and bidding on their behalf. This behavior leads to well-structured profit functions for the sellers. The second factor is that sellers have identical costs. Many sellers in online consumer-to-consumer markets are reselling previously bought items, so one would not expect significant variability in the costs of sellers.

To conclude, we briefly consider the case of online auctions but where the two sellers simultaneously choose their prices. This case differs only from Burguet and Sákovics (1999) in the assumption that buyers can freely buy from either seller rather than commit to one seller's auction or the other's. It is straightforward to show using our profit functions (and similar to our proof of Lemma 2) that no pure strategy Nash equilibrium exists. Furthermore, it can be shown that the support of any mixed strategy equilibrium must be identical but cannot include zero, the same result as Burguet and Sákovics (1999) obtain when buyers commit to one auction or the other. Further, using a standard argument, the support of prices that a seller mixes over in any mixed strategy equilibrium must not contain any gaps or atoms. It can be shown that the supremum of the support of each seller's mixed strategy equals r_2^* — the price the second-moving seller chooses in the equilibrium in the version of the game when sellers choose reserve prices sequentially. If it were lower, then any seller pricing near enough this supremum would be the high priced seller with probability near enough one that it would be profitable to deviate to r_2^*. Because sellers must be indifferent between any prices that they mix over, this means that in a mixed strategy equilibrium each seller must earn (arbitrarily close to) $\pi_2(r_2^*)$, as choosing a price nearly r_2^* would result in the highest price with probability nearly one. It follows given our earlier work in this paper, that each seller would then have an incentive for some strategic move in which he commits to a reserve price prior to the other seller, thereby making the assumption of simultaneous choice of reserve prices fragile.

Figure 1: Equilibrium in reserve prices (points A and B) when $n = 3$ and $v_i \sim U[0, 1]$.



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