# Quantitative Easing and Direct Lending in Response to the COVID-19 Crisis 

Filippo Occhino ${ }^{\dagger}$

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#### Abstract

This paper develops a dynamic general equilibrium model to study quantitative easing (QE) and direct lending to firms. In the model, QE works through three channels: The expansion of bank reserves raises liquidity and lowers the liquidity premium; The purchase of assets withdraws risk and lowers the volatility risk premium; And the resulting economic stimulus lowers the credit risk premium. Since the level of bank reserves was greater in 2020 than in 2008, the liquidity premium channel was weaker, and QE was less expansionary. A QE program worth 4 percent of GDP would have expanded output by 3.1 and 0.5 percent in 2008 and 2020, respectively. Direct lending to firms is more expansionary than QE because it substitutes bank lending and mitigates the credit risk frictions associated with bank lending. In contrast, QE stimulates bank lending and worsens the frictions. A direct lending program worth 4 percent of GDP would have expanded output by 3.4 and 0.8 percent in 2008 and 2020, respectively.


Keywords: Bank reserves, liquidity premium, risk premium, interest rate spread.

JEL Classification Numbers: E32, E43, E51, E52, E58.

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## 1 Introduction

The COVID-19 crisis cut off firms' cash flow and available funds, threatening the survival of many firms. The Federal Reserve responded with numerous programs, including quantitative easing (QE) and direct lending to firms, to prevent a collapse in firms' available funds. This paper develops a dynamic general equilibrium model to evaluate various channels through which these programs work.

QE refers to the Federal Reserve's large-scale purchases of Treasury bonds and other long-term securities financed by increased bank reserves. In March 2020, the Federal Reserve announced purchases of at least $\$ 500$ billion in Treasuries and $\$ 200$ billion in agency mortgage-backed securities totaling 3.3 percent of 2020 GDP. At the end of the same month, it modified the announcement, making the purchases openended as needed to support market functioning and monetary policy transmission. In June 2020, it announced purchases of at least $\$ 80$ billion in Treasuries and $\$ 40$ billion in agency mortgage-backed securities per month. For comparison, the first announced QE in November 2008 consisted of purchases of up to $\$ 600$ billion in agency debt and mortgage-backed securities worth 4 percent of 2008 GDP.

The Federal Reserve also introduced new programs to lend directly to firms. In March 2020, it announced purchases of newly issued investment-grade corporate bonds and loans through the Primary Market Corporate Credit Facility. The purchase price was informed by market conditions plus a 100bps facility fee. In April 2020, the Federal Reserve announced loans to small and mid-size businesses through various Main Street lending facilities. The loans were for five years at LIBOR plus 3 percent, with interest payment and principal repayment deferred for one and two years, respectively.

This paper studies how QE works and finds that it was much less expansionary in 2020 than in 2008. Treasury bond purchases worth 4 percent of GDP would have raised real GDP by 3.1 and 0.5 percent in 2008 and 2020, respectively. The reason why QE was less expansionary in 2020 has to do with the level of bank reserves. QE
works by expanding bank reserves and decreasing the net supply of Treasury bonds. According to the model, the most important channel through which it works is the expansion of bank reserves, which lowers the liquidity premium and the loan-deposit spread and stimulates bank lending, firms' investment, and output. The strength of this channel depends on the level of bank reserves before QE: The greater the level of bank reserves, the smaller the effect of QE on the liquidity premium and output. Since the level of bank reserves was greater in 2020 than in 2008, QE had a smaller effect.

The paper also examines two channels through which direct lending to firms has more expansionary effects than QE. First, direct lending withdraws more risk from the private sector than QE because QE decreases Treasury bonds while direct lending decreases riskier bank loans. Second, QE stimulates bank lending while direct lending substitutes it, so QE worsens the financial frictions associated with firms' borrowing from the private sector while direct lending mitigates them-A channel put forth by Sims and Wu (2020) and Cardamone, Sims, and Wu (2023). According to the model, the first channel is small, while the second is larger. A direct lending program worth 4 percent of GDP would have raised real GDP by 3.4 percent in 2008 and 0.8 percent in 2020, 0.3 percentage points more than QE.

In the model, banks accept household demand deposits and hold reserves, Treasury bills, Treasury bonds, and firms' loans. Bank assets differ in liquidity, volatility risk, and credit risk, so they are not perfect substitutes and earn different rates of return. Less liquidity and greater risk induce a smaller asset demand, a lower price, and a higher return. Less liquid assets earn a higher liquidity premium and rate of return, and riskier assets earn a higher risk premium and rate of return.

Liquidity. Deposits are on demand, and reserves are the only perfectly liquid assets to meet deposit withdrawal requests. I model banks' need for reserves using a penalty function that increases with the ratio of deposits to reserves and makes reserves imperfect substitutes for other bank assets. Since non-reserve assets are less liquid than deposits, the rate of return of non-reserve assets is
greater than the deposit rate in equilibrium. The spread equals a liquidity premium that increases with the deposit-reserve ratio. Here, I model a narrow liquidity aspect, the ability of an asset to serve as money and meet deposit withdrawal requests. Without modeling it explicitly, I aim at capturing the flow of households' withdrawals and deposits within each period and the need to hold a fraction of deposits as reserves to meet any positive net withdrawals. I do not model how assets differ in market liquidity, that is, how easy and cheap selling assets at the beginning of each period is and how asset prices deviate from fundamentals because of market liquidity.

Volatility risk. Reserves and Treasury bills held across periods are risk-free, while the value of Treasury bonds and firms' loans is volatile. Volatility risk can be due to various underlying sources of price volatility, including duration risk andin the case of loans-credit risk. Another underlying source is market liquidity risk, the price volatility associated with market liquidity. Here, "liquidity" refers to a broader aspect than the one defined in the previous paragraph. For risk management, banks need to hold enough equity to cover losses in their portfolio of volatile assets. I model banks' need for equity using a penalty function that increases with the ratio of risk-weighted assets to bank equity. Risk-weighted assets are computed using a larger risk weight for loans than bonds. The penalty function makes bonds and loans imperfect substitutes for bills. Since bonds and loans are volatile while bills are risk-free, the rates of return of bonds and loans are greater than the Treasury bill rate. The spread equals a volatility risk premium that increases with the ratio of risk-weighted assets to bank equity. Implicitly, the volatility risk premium incorporates components for duration risk, credit risk, and market liquidity risk.

Credit risk. Unlike Treasuries, firms' loans are subject to credit risk. Firms can default on their loans, and the default probability drives a wedge between the lending rate paid by firms and the rate of return expected by banks. The wedge
creates a credit risk premium that raises the lending rate above the rate of return of assets with no credit risk. I assume that the credit risk premium is negatively related to aggregate output and positively related to the ratio of firms' borrowing from banks to cash flow. The latter dependence captures the firm-level financial frictions modeled by Sims and Wu (2020) and Cardamone, Sims, and Wu (2023).

The different liquidity and risk characteristics of deposits and loans create a spread between the lending and deposit rates. The loan-deposit spread is the sum of the three premiums: the liquidity premium, the volatility risk premium, and the credit risk premium. Federal Reserve programs work by reducing all three premiums. As the premiums decrease, the loan-deposit spread decreases and stimulates firms' investment and output.

1. The expansion of bank reserves raises liquidity and lowers the liquidity premium. This effect depends on the imperfect substitutability between reserves and other bank assets. Bernanke and Reinhart (2004) point out that the view that an increase in money supply leads investors to rebalance their portfolio and lowers not only short-term but also long-term interest rates has a long intellectual history, including Brainard and Tobin (1968) and Tobin (1969). Andrés, López-Salido, and Nelson (2004) model this effect in a dynamic general equilibrium model with imperfect asset substitutability. Christensen and Krogstrup (2022) propose a model where assets are imperfect substitutes, the market for reserves is segmented, and increased bank reserves induce portfolio balance effects and lower long-term yields. Christensen and Krogstrup (2019) provide event-study evidence that an increase in bank reserves per se lowers long-term yields significantly. More specifically, they show that announcements by the Swiss National Bank in August 2011 to expand bank reserves without purchasing long-term securities lowered term premiums and long-term yields. Krogstrup, Reynard, and Sutter (2012) use time-series regressions to show that
the Fed expansion of reserves in 2009-2011 lowered the liquidity premium and long-term interest rates.
2. QE reduces the net supply of Treasury bonds, while direct lending reduces firms' demand for bank loans. In both cases, banks' holdings of volatile assets decrease and lower the volatility risk premium. This channel depends on the imperfect substitutability of non-reserve assets and is the portfolio balance channel modeled by Vayanos and Vila (2009) and emphasized by the literature.
3. The economic stimulus generated by the first two channels lowers the credit risk premium. This channel is an amplification mechanism that works with expansionary policies.

As evident from this initial discussion, this paper studies some, but not all, Fed programs' mechanisms. While the volatility risk premium implicitly incorporates a component for market liquidity risk, the paper does not explicitly model market functioning and market liquidity, so it may underestimate the effects of Fed programs on market liquidity, market functioning, and output. Also, the paper does not model the signaling effects of Fed programs on expected future conventional monetary policy and short-term policy rates.

Furthermore, the model focuses on the effects on banks and firms, not households, so it does not capture the expansionary effects through the housing channel and, more generally, the household side of the economy. Purchases of Treasuries and agency mortgage-backed securities lower mortgage premiums, spreads, and rates, stimulating spending in the housing sector (Hancock and Passmore 2014).

Finally, the paper studies the effects of the unanticipated introduction of Fed programs. However, after the first QE announcement in 2008, the public began to anticipate the introduction of Fed programs in response to crises, so the 2020 programs were partly anticipated. A rule describing the introduction of Fed programs in response to crises would change agents' behavior and have different effects from unanticipated policy changes. Results would depend on the extent agents anticipate
the introduction of old programs, like QE, or new programs, like direct lending. This paper sheds light only on the unanticipated part of policy changes in 2008 and 2020.

## Related literature

This paper builds upon the seminal work of Sims and Wu (2020) and its published version, Cardamone, Sims, and Wu (2023). The motivation and some modeling features follow from their work. Next, I describe the differences between our mechanisms and compare them quantitatively.

We both conclude that QE was less effective in response to the COVID-19 crisis than the Great Recession but for different reasons. They emphasize that, during the Great Recession, the amount of loans given by financial intermediaries (banks) to firms was limited by financial constraints faced by banks. In such a situation, QE and direct lending could relax the bank-level constraints and stimulate the economy. In contrast, during the COVID-19 crisis, the amount of loans given by banks to firms was limited by financial constraints faced by firms: Firms were not able to obtain bank loans because their cash flow collapsed. In such a situation, QE could not relax the firm-level constraints and was ineffective, while direct lending could relax them and stimulate the economy.

In my model, the main channel through which Fed programs work is the expansion of bank reserves, which lowers the liquidity premium. If the initial level of bank reserves is already high, the additional increase in bank reserves has a smaller effect, and this channel is weaker. Since bank reserves were greater in 2020 than in 2008, QE and direct lending were less effective in response to the COVID-19 crisis than the Great Recession. The decrease in effectiveness is quantitatively tied to the empirical increase in bank reserves.

Our two mechanisms can work together and offer two reasons why QE was less effective in response to the COVID-19 crisis than the Great Recession. Within the context of my model, my mechanism is responsible for a greater decrease in QE effectiveness: The output response to QE was 0.5 and 2.6 percentage points smaller
in 2020 than in 2008 because of their mechanism and mine, respectively.
Our two interpretations also differ regarding the QE effectiveness between 2008 and 2020. In their model, the decrease in effectiveness depends on the worsening of firm-level financial frictions in 2020. In contrast, my model suggests that QE became less effective before the COVID-19 crisis since bank reserves rose after 2008. According to my model, because of the increase in bank reserves announced in 2008, later QE programs became 50 percent less expansionary. My mechanism, then, offers a possible reason why event studies tend to find that the announcements of later rounds of QE in 2010 and 2012 had smaller effects than the announcements of the first round in 2008-2009. This reason adds to the explanations proposed by the literature that the later rounds were better anticipated and financial conditions were less strained ( Kr ishnamurthy and Vissing-Jorgensen 2013, Cahill et al. 2013, Bernanke 2020, D'Amico and Seida 2023.)

In my model, direct lending is more expansionary than QE for the reason they describe: QE stimulates bank lending, while direct lending substitutes it, so QE worsens the financial frictions associated with firms' borrowing from the private sector, while direct lending mitigates them. With plausible parameter values, this mechanism is sizeable but much smaller than shown in their paper. A direct lending program worth 4 percent of GDP would have raised output by 0.3 percentage points more than QE in 2020, according to my model, while it would have raised output by 6 percentage points more, according to their paper.

More generally, my paper contributes to the literature that studies the macroeconomic effects of QE using dynamic general equilibrium models with financial frictions and segmented asset markets, such as Gertler and Karadi (2011, 2013), Chen, Cúrdia, and Ferrero (2012), Carlstrom, Fuerst, and Paustian (2017), Sims and Wu (2021), and Bordo and Sinha (2023).

The paper also contributes to the literature that estimates the effects of QE. A few papers use a Bayesian VAR methodology to estimate the effect on real GDP. For instance, Weale and Wieladek (2016) find that central bank purchases of government
bonds worth 1 percent of GDP raise real GDP by about 0.6 percent, while Baumeister and Benati (2013) find that the first round of QE raised real GDP by about 3.5 percent in the first quarter of 2019. While these papers estimate the effects of past rounds of QE, my paper focuses on the decreased effectiveness of QE and other Fed programs over time.

Several papers, including Gagnon et al. (2011), D'Amico et al. (2012), D'Amico and King (2013), Hamilton and Wu (2012), Neely (2015), and Swanson (2021) focus on the announcement effects on asset prices and interest rates. Krishnamurthy and Vissing-Jorgensen (2011) study separately several channels through which QE affects interest rates, including some related to the ones I model in this paper: a liquidity channel, a duration risk channel, and a default risk channel. Greenlaw et al. (2018) provide evidence that the effect of QE on interest rates may be smaller, less persistent, and more uncertain than estimated by the event-study literature. Event studies identify some causal effects of programs' announcements but miss the effects that depend on programs' implementation over the years and are not captured by asset prices and interest rates at the time of announcements, like the effects of changes in the balance sheets of banks and firms on premiums described in this paper.

In the rest of the paper, Section 2 describes the model; Section 3 details the calibration; Section 4 explains the results; and Section 5 concludes.

## 2 Model

The model features households, banks, firms, a government, and the central bank. Households supply labor to firms and hold deposits at banks. Banks hold reserves at the central bank, purchase short-term and long-term government debt, and lend to firms. Firms invest and produce.

Three financial frictions discourage firms' investment and output. Banks face a liquidity friction and a volatility risk friction, while firms face a credit risk friction. The frictions generate three premiums and create a spread between the firms' bor-
rowing rate and the household deposit rate.

### 2.1 Households

Households consume $c_{t}^{H}$, supply labor, $n_{t}$, and receive wages, $w_{t} n_{t}$. They deposit $D_{t+1}^{H}$ at banks, receive gross-of-interest deposit repayments from banks, $\left(1+r_{t}^{D}\right) D_{t}^{H}$, and lump-sum transfers from the government, $T_{t}$. The households' budget constraint is:

$$
\begin{equation*}
c_{t}^{H}+D_{t+1}^{H}=w_{t} n_{t}+\left(1+r_{t}^{D}\right) D_{t}^{H}+T_{t} \tag{1}
\end{equation*}
$$

The households' optimization problem is:

$$
\begin{gather*}
\max _{\left\{c_{t}^{H}, n_{t}, D_{t+1}^{H}\right\}_{t=0}^{\infty}} E_{0} \sum_{t=0}^{\infty} \hat{\beta}^{t}\left[u\left(c_{t}^{H}\right)-v\left(n_{t}\right)\right]  \tag{2}\\
\text { subject to (1), }
\end{gather*}
$$

where $\hat{\beta} \in(0,1)$ is the households' discount factor, $u(c)$ is such that $u^{\prime}(c) \equiv c^{-\gamma}, \gamma>0$ is the relative risk aversion, $v(n) \equiv \Phi n^{1+1 / \varphi}, \Phi>0, \varphi>0$ is the Frisch elasticity of labor supply, and $E_{0}$ is the expectation operator.

The first-order conditions are:

$$
\begin{align*}
\frac{v^{\prime}\left(n_{t}\right)}{u^{\prime}\left(c_{t}^{H}\right)} & =w_{t}  \tag{3}\\
1 & =E_{t}\left\{\frac{\hat{\beta} u^{\prime}\left(c_{t+1}^{H}\right)}{u^{\prime}\left(c_{t}^{H}\right)}\left(1+r_{t+1}^{D}\right)\right\} . \tag{4}
\end{align*}
$$

### 2.2 Banks

Banks receive household deposits, $D_{t+1}$, hold reserves at the central bank, $R_{t+1}$, shortterm Treasury bills, $M_{t+1}$, and long-term Treasury bonds, $N_{t+1}$, and make loans to firms, $L_{t+1}$. Treasury bonds are modeled as perpetuities with decaying coupon payments, as in Cardamone, Sims, and Wu (2023). Let $\kappa \in(0,1)$ denote the decay
parameter for coupon payments; that is, the coupon is equal to 1 in the first period, $\kappa$ in the second period, $\kappa^{2}$ in the third period, and so on. Equivalently, one can think of a bond as a promise to repay the next period a unitary coupon plus a fraction $\kappa$ of a new bond. Let $q_{t}$ be the price of a bond, and let

$$
\begin{equation*}
B_{t+1} \equiv M_{t+1}+q_{t} N_{t+1} \tag{5}
\end{equation*}
$$

be the value of government debt held by banks.
Banks face two financial frictions. The first one models banks' need for reserves and how bank assets differ in liquidity. Deposits are on-demand liabilities, and banks need reserves to meet any positive net demand to withdraw. It may help to think that all assets can be traded across periods, but only deposits and reserves are liquid within periods. To manage the deposit withdrawal risk and meet any regulatory constraints on required reserves, banks need to hold part of their deposits as reserves at the central bank. I model the special role of reserves with a penalty function that increases with the ratio of deposits to reserves:

$$
\begin{equation*}
g_{t+1}=A_{g} \frac{1}{\lambda}\left(\frac{D_{t+1}}{R_{t+1}} \frac{R}{D}\right)^{\lambda} \tag{6}
\end{equation*}
$$

where $D$ and $R$ are steady-state values, $A_{g}>0$, and $\lambda>1$. This penalty function can be seen as a smoother and more flexible alternative to a minimum ratio of reserves to deposits. The friction creates a liquidity premium that drives down the return of deposits and reserves relative to illiquid assets, such as Treasuries and bank loans.

The second financial friction models banks' need for equity and how bank assets differ in volatility risk. The value of long-term Treasury bonds and loans changes because of changes in interest rates and credit risk, respectively. Banks need to maintain a capital cushion to absorb any larger-than-expected drop in asset values. To manage the risk and meet all regulatory constraints on risk-weighted assets, banks need to hold enough equity relative to their risk-weighted assets. Let bank equity,
$V_{t+1}$, be the difference between bank assets and liabilities,

$$
\begin{equation*}
V_{t+1} \equiv R_{t+1}+M_{t+1}+q_{t} N_{t+1}+L_{t+1}-D_{t+1} \tag{7}
\end{equation*}
$$

and let banks' risk-weighted assets, $Z_{t+1}$, be the weighted sum of long-term Treasury and bank loans,

$$
\begin{equation*}
Z_{t+1} \equiv \omega_{N} q_{t} N_{t+1}+\omega_{L} L_{t+1} \tag{8}
\end{equation*}
$$

where $\omega_{N} \in(0,1)$ and $\omega_{L} \in(0,1)$ are the risk weights of bonds and loans, respectively. I model the banks' need for equity using a penalty function that increases with the ratio of their risk-weighted assets to equity:

$$
\begin{equation*}
h_{t+1}=A_{h} \frac{1}{\rho}\left(\frac{Z_{t+1}}{V_{t+1}} \frac{V}{Z}\right)^{\rho} \tag{9}
\end{equation*}
$$

where $Z$ and $V$ are steady-state values, $A_{h}>0$, and $\rho>1$. This penalty function is smoother and more flexible than a minimum ratio of equity to risk-weighted assets. The friction creates a volatility risk premium that drives up the return of volatile assets, such as long-term Treasury bonds and bank loans, relative to risk-free ones, such as deposits, reserves, and Treasury bills.

Banks make their portfolio choices taking into account how the penalties, $g_{t+1}$ and $h_{t+1}$, depend on their portfolio choices. Then, the equilibrium values of the penalties, denoted by $\bar{g}_{t+1}$ and $\bar{h}_{t+1}$, are rebated lump-sum to the banks, so banks take these two equilibrium values as given while choosing their portfolio.

The bank's constraint is:

$$
\begin{align*}
& c_{t}+R_{t+1}+M_{t+1}+q_{t} N_{t+1}+L_{t+1}-D_{t+1}+g_{t}-\bar{g}_{t}+h_{t}-\bar{h}_{t}= \\
& \quad\left(1+r_{t}^{R}\right) R_{t}+\left(1+r_{t}^{M}\right) M_{t}+\left(1+\kappa q_{t}\right) N_{t}+\left(1+r_{t}^{L}\right) L_{t}-\left(1+r_{t}^{D}\right) D_{t} . \tag{10}
\end{align*}
$$

On the left-hand side, the first term represents consumption expenditures. The next
four terms are the bank purchases of assets (reserves, Treasury bills, Treasury bonds, and loans). The sixth term represents the funds received from depositors. The final four terms are the penalties associated with the frictions minus their equilibrium values. On the right-hand side, the first four terms are the gross-of-interest payoffs from bank asset purchases in the previous period, while the last term is the gross-ofinterest payoff paid to depositors.

The optimization problem solved by the owner of a bank is:

$$
\begin{equation*}
\max _{\left\{c_{t}, D_{t+1}, R_{t+1}, M_{t+1}, N_{t+1}, L_{t+1}, g_{t+1}, h_{t+1}\right\}_{t=0}^{\infty}} E_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(c_{t}\right) \tag{11}
\end{equation*}
$$

subject to (6), (9), and (10),
where $\beta \in(0,1)$ is the banks' discount factor and $u(c)$ is the same function as the one for households.

The first-order conditions are:

$$
\begin{align*}
& 1=E_{t}\left\{\frac{\beta u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)}\left(1+r_{t+1}^{M}+\rho h_{t+1} / V_{t+1}\right)\right\}  \tag{12}\\
& 1=E_{t}\left\{\frac{\beta u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)}\left(1+r_{t+1}^{N}+\rho h_{t+1} / V_{t+1}-\omega_{N} \rho h_{t+1} / Z_{t+1}\right)\right\}  \tag{13}\\
& 1=E_{t}\left\{\frac{\beta u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)}\left(1+r_{t+1}^{L}+\rho h_{t+1} / V_{t+1}-\omega_{L} \rho h_{t+1} / Z_{t+1}\right)\right\}  \tag{14}\\
& 1=E_{t}\left\{\frac{\beta u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)}\left(1+r_{t+1}^{R}+\rho h_{t+1} / V_{t+1}+\lambda g_{t+1} / R_{t+1}\right)\right\}  \tag{15}\\
& 1=E_{t}\left\{\frac{\beta u^{\prime}\left(c_{t+1}\right)}{u^{\prime}\left(c_{t}\right)}\left(1+r_{t+1}^{D}+\rho h_{t+1} / V_{t+1}+\lambda g_{t+1} / D_{t+1}\right)\right\} \tag{16}
\end{align*}
$$

where

$$
\begin{equation*}
1+r_{t+1}^{N} \equiv \frac{1+\kappa q_{t+1}}{q_{t}} \tag{17}
\end{equation*}
$$

is the stochastic gross rate of return on Treasury bonds. From the first-order condi-
tions, one can derive the following first-order linear approximations for the spreads:

$$
\begin{align*}
& r_{t+1}^{N}-r_{t+1}^{M} \approx \omega_{N} \rho h_{t+1} / Z_{t+1}=\omega_{N} A_{h} \frac{Z_{t+1}^{\rho-1}}{V_{t+1}^{\rho}} \frac{V^{\rho}}{Z^{\rho}}  \tag{18}\\
& r_{t+1}^{L}-r_{t+1}^{M} \approx \omega_{L} \rho h_{t+1} / Z_{t+1}=\omega_{L} A_{h} \frac{Z_{t+1}^{\rho-1}}{V_{t+1}^{\rho}} \frac{V^{\rho}}{Z^{\rho}}  \tag{19}\\
& r_{t+1}^{M}-r_{t+1}^{R} \approx \lambda g_{t+1} / R_{t+1}=A_{g} \frac{D_{t+1}^{\lambda}}{R_{t+1}^{\lambda-1}} \frac{R^{\lambda}}{D^{\lambda}}  \tag{20}\\
& r_{t+1}^{M}-r_{t+1}^{D} \approx \lambda g_{t+1} / D_{t+1}=A_{g} \frac{D_{t+1}^{\lambda-1}}{R_{t+1}^{\lambda}} \frac{R^{\lambda}}{D^{\lambda}} \tag{21}
\end{align*}
$$

The first two equations show that the rates of return on Treasury bonds and bank loans increase relative to the Treasury bill rate as risk-weighted assets increase or bank equity decreases. The next two equations show that the deposit and reserve rates decrease relative to the Treasury bill rate as deposits increase or reserves decrease.

### 2.3 Firms

Firms begin period $t$ with capital, $k_{t}$, and loans, $L_{t}^{F}$, which are the sum of private bank lending, $L_{t}$, and central bank direct lending, $\tilde{L}_{t}$. They hire labor, $l_{t}$, at the wage rate, $w_{t}$, and produce output,

$$
\begin{equation*}
y_{t} \equiv \theta_{t} A f\left(k_{t}, l_{t}\right), \tag{22}
\end{equation*}
$$

where $A>0, \theta_{t}$ is a stationary stochastic productivity variable with a mean equal to one, $f(k, l) \equiv k^{\alpha} l^{1-\alpha}$, and $\alpha \in(0,1)$.

Firms invest $x_{t}$, so capital evolves according to:

$$
\begin{equation*}
k_{t+1}=(1-\delta) k_{t}+x_{t}-\frac{\psi}{2}\left(\frac{k_{t+1}-k_{t}}{k}\right)^{2} k \tag{23}
\end{equation*}
$$

where $\delta \in(0,1)$ is the economic depreciation rate, the last term is a capital adjustment cost, $\psi>0$, and $k>0$ is the steady-state level of capital.

Firms face a financial friction that models how credit risk raises the cost of external
funds. Credit risk drives a wedge between the lending rate paid by firms and the rate expected by creditors. To simplify, if the default probability is 1 percent, the expected rate of return is 1 percentage point less than the lending rate. I model credit risk assuming that the interest rate paid by firms, $\tilde{r}_{t+1}^{L}$, is higher than the interest rate received by creditors, $r_{t+1}^{L}$. The wedge between the two rates, $z_{t+1}$, works like a tax and discourages lending. The friction creates a credit risk premium that drives up firms' cost of external funds and user cost of capital relative to the return of assets with no credit risk.

Since default probabilities and credit risk are countercyclical, I assume that the wedge decreases as aggregate output increases. I also assume that the wedge increases with the ratio of bank lending to firms' cash flow:

$$
\begin{equation*}
z_{t+1} \equiv \tilde{r}_{t+1}^{L}-r_{t+1}^{L}=A_{z}-\eta_{y} \log \left(\frac{y_{t}}{y}\right)+\eta_{L} \log \left(\frac{L_{t}}{C F_{t}} \frac{C F}{L}\right) \tag{24}
\end{equation*}
$$

where $C F_{t} \equiv y_{t}-w_{t} l_{t}$ is firms' cash flow defined as in Cardamone, Sims, and $\mathrm{Wu}(2023) ; A_{z}>0, \eta_{y}>0$, and $\eta_{L}>0$ are positive constants; and $y>0, L>0$, and $C F>0$ denote the steady-state levels of aggregate output, bank loans, and cash flow, respectively.

The dependence of the wedge on the ratio of bank lending to firms' cash flow captures financial frictions that constrain firms' borrowing from the private sector, like the firm-level financial friction modeled by Cardamone, Sims, and Wu (2023). They assume that a firm's borrowing from the private sector is constrained by the firm's current cash flow, so bank lending is constrained while central bank direct lending is not. Since QE works by stimulating bank lending, it is less expansionary than direct lending. Similarly, in my model, the dependence of the wedge on bank lending constrains firms' borrowing from banks, $L_{t}$, but not from the central bank, $\tilde{L}_{t}$, making QE less expansionary than direct lending.

The equilibrium value of the difference between the return paid by firms and the return received by the creditors equals $z_{t+1} \bar{L}_{t+1}^{F}$, where $\bar{L}_{t+1}^{F}$ denotes the equilibrium
value of loans. The equilibrium value of the difference is rebated lump-sum to the firms, so firms take it as given while choosing their loans.

The firm's budget constraint is:

$$
\begin{equation*}
c_{t}^{F}+\left(1+\tilde{r}_{t}^{L}\right) L_{t}^{F}-z_{t} \bar{L}_{t}^{F}=y_{t}-w_{t} l_{t}-x_{t}+L_{t+1}^{F} \tag{25}
\end{equation*}
$$

The left-hand side lists the firm's consumption expenditure, the gross-of-interest loans repaid to the creditors, and the lump-sum rebate. The right-hand side lists the firm's revenue net of wages and investment expenditures, and the newly borrowed funds.

The firm's total available funds, $Q_{t}$, are defined as all funds available to finance consumption and investment:

$$
\begin{equation*}
Q_{t} \equiv y_{t}-w_{t} l_{t}+L_{t+1}^{F}-\left(1+\tilde{r}_{t}^{L}\right) L_{t}^{F}+z_{t} \bar{L}_{t}^{F}=c_{t}^{F}+x_{t} \tag{26}
\end{equation*}
$$

The optimization problem solved by the owner of a firm is:

$$
\begin{equation*}
\max _{\left\{c_{t}^{F}, L_{t+1}^{F}, y_{t}, l_{t}, x_{t}, k_{t+1}\right\}_{t=0}^{\infty}} E_{0} \sum_{t=0}^{\infty} \tilde{\beta}^{t} u\left(c_{t}^{F}\right) \tag{27}
\end{equation*}
$$

subject to (22), (23), and (25),
where $\tilde{\beta} \in(0,1)$ is the firms' discount factor, and $u(c)$ is the same function as the one for households and banks.

The first-order conditions are:

$$
\begin{align*}
\theta_{t} A f_{l}\left(k_{t}, l_{t}\right) & =w_{t}  \tag{28}\\
1+\psi \frac{k_{t+1}-k_{t}}{k} & =E_{t}\left\{\frac{\tilde{\beta} u^{\prime}\left(c_{t+1}^{F}\right)}{u^{\prime}\left(c_{t}^{F}\right)}\left[1-\delta+\psi \frac{k_{t+2}-k_{t+1}}{k}+\theta_{t+1} A f_{k}\left(k_{t+1}, l_{t+1}\right)\right]\right\}  \tag{29}\\
1 & =E_{t}\left\{\frac{\tilde{\beta} u^{\prime}\left(c_{t+1}^{F}\right)}{u^{\prime}\left(c_{t}^{F}\right)}\left(1+\tilde{r}_{t+1}^{L}\right)\right\} . \tag{30}
\end{align*}
$$

### 2.4 Central bank

The central bank accepts bank reserves, holds Treasury bills and bonds, and makes direct loans to firms:

$$
\begin{equation*}
R_{t+1}=\tilde{M}_{t+1}+q_{t} \tilde{N}_{t+1}+\tilde{L}_{t+1} \tag{31}
\end{equation*}
$$

Let

$$
\begin{equation*}
\tilde{B}_{t+1} \equiv \tilde{M}_{t+1}+q_{t} \tilde{N}_{t+1} \tag{32}
\end{equation*}
$$

be the value of government debt held by the central bank.
To model QE, I assume that the central-bank-held Treasury bonds, $\tilde{N}_{t+1}$, follow the first-order autoregressive process:

$$
\begin{equation*}
\tilde{N}_{t+1}-\tilde{N}=\rho_{Q E}\left(\tilde{N}_{t}-\tilde{N}\right)+\epsilon_{Q E, t+1} \tag{33}
\end{equation*}
$$

where $\tilde{N}>0$ is the steady-state value, $\rho_{Q E} \in(0,1)$, and $\epsilon_{Q E, t+1}$ is a QE policy shock distributed as a normal random variable with a mean equal to zero. The last three equations indicate that a positive QE policy shock persistently raises the central-bank-held Treasury bonds, $\tilde{N}_{t+1}$, the central-bank-held government debt, $\tilde{B}_{t+1}$, and bank reserves, $R_{t+1}$.

Similarly, to model a direct lending program, I assume that central bank loans, $\tilde{L}_{t+1}$, which are equal to zero in the steady state, follow the first-order autoregressive process:

$$
\begin{equation*}
\tilde{L}_{t+1}=\rho_{D L} \tilde{L}_{t}+\epsilon_{D L, t+1} \tag{34}
\end{equation*}
$$

where $\rho_{D L} \in(0,1)$ and $\epsilon_{D L, t+1}$ is a direct-lending policy shock distributed as a normal random variable with a mean equal to zero. A positive direct-lending policy shock raises persistently central bank loans, $\tilde{L}_{t+1}$, and bank reserves, $R_{t+1}$.

The central bank gives the government the seigniorage, $S_{t}$, which is the difference between the returns of its assets and liabilities:

$$
\begin{equation*}
S_{t}=\left(1+r_{t}^{M}\right) \tilde{M}_{t}+\left(1+\kappa q_{t}\right) \tilde{N}_{t}+\left(1+r_{t}^{L}\right) \tilde{L}_{t}-\left(1+r_{t}^{R}\right) R_{t} . \tag{35}
\end{equation*}
$$

### 2.5 Government

The government sells and redeems Treasury bills and bonds, spends a constant $G>0$, receives the seigniorage, $S_{t}$, from the central bank, and distributes lump-sum transfers to households, $T_{t}$ :

$$
\begin{equation*}
\hat{M}_{t+1}+q_{t} \hat{N}_{t+1}=\left(1+r_{t}^{M}\right) \hat{M}_{t}+\left(1+\kappa q_{t}\right) \hat{N}_{t}+G-S_{t}+T_{t} . \tag{36}
\end{equation*}
$$

It uses the lump-sum transfers to balance its intertemporal budget constraint. I assume that the lump-sum transfers respond to changes in government debt enough to ensure that government debt is stationary and an equilibrium exists:

$$
\begin{equation*}
T_{t}=A_{T}-\tau r^{M}\left(\hat{M}_{t+1}+q_{t} \hat{N}_{t+1}\right), \tag{37}
\end{equation*}
$$

where $A_{T}$ is a constant, $r^{M}$ is the steady-state Treasury bill rate, and $\tau>0$.

### 2.6 Equilibrium conditions

Let

$$
\begin{equation*}
C_{t} \equiv c_{t}^{H}+c_{t}+c_{t}^{F} \tag{38}
\end{equation*}
$$

be aggregate private consumption. The equilibrium condition for the goods market equates the demand for private consumption, government consumption, and invest-
ment to production:

$$
\begin{equation*}
C_{t}+G+x_{t}=y_{t} . \tag{39}
\end{equation*}
$$

The remaining equilibrium conditions equate demand and supply in the markets for labor, deposits, Treasury bills, Treasury bonds, and loans:

$$
\begin{align*}
l_{t} & =n_{t}  \tag{40}\\
D_{t+1} & =D_{t+1}^{H}  \tag{41}\\
M_{t+1}+\tilde{M}_{t+1} & =\hat{M}_{t+1}  \tag{42}\\
N_{t+1}+\tilde{N}_{t+1} & =\hat{N}_{t+1}  \tag{43}\\
L_{t+1}^{F} & =L_{t+1}+\tilde{L}_{t+1} \tag{44}
\end{align*}
$$

One variable that plays a crucial role is the loan-deposit spread, $s_{t+1}$, the spread between the rate paid by firms, $\tilde{r}_{t+1}^{L}$, and the rate received by depositors, $r_{t+1}^{D}$. A large spread discourages firms' investment and output. Using equations (19), (21), and (24), one can decompose the spread into the sum of three premiums:

$$
\begin{align*}
& s_{t+1} \equiv \tilde{r}_{t+1}^{L}-r_{t+1}^{D} \\
& =\left(\tilde{r}_{t+1}^{L}-r_{t+1}^{L}\right)+\left(r_{t+1}^{L}-r_{t+1}^{M}\right)+\left(r_{t+1}^{M}-r_{t+1}^{D}\right) \\
& \approx \underbrace{\left[A_{z}-\eta_{y} \log \left(\frac{y_{t}}{y}\right)+\eta_{L} \log \left(\frac{L_{t}}{C F_{t}} \frac{C F}{L}\right)\right]}_{\text {credit risk premium } \pi_{t+1}^{\text {cred }}}+\underbrace{\omega_{L} A_{h} \frac{Z_{t+1}^{\rho-1}}{V_{t+1}^{\rho}} \frac{V^{\rho}}{Z^{\rho}}}_{\text {vol. risk premium } \pi_{t+1}^{v o l}}+\underbrace{A_{g} \frac{D_{t+1}^{\lambda-1}}{R_{t+1}^{\lambda}} \frac{R^{\lambda}}{D^{\lambda}}}_{\text {liq. premium } \pi_{t+1}^{\text {liq }}} . \tag{45}
\end{align*}
$$

The term in square brackets on the right-hand side is a credit risk premium that decreases with aggregate output and increases with the ratio of bank lending to firms' cash flow. The next term is a volatility risk premium that increases with risk-weighted assets (long-term Treasury bonds and loans) and decreases with bank equity. The final term is a liquidity premium that increases with bank deposits and
decreases with reserves. By changing the net supply of assets with different liquidity and risk characteristics, Fed programs can reduce these premiums and stimulate firms' investment and output.

## 3 Calibration

In this section, I first describe the parameter setting for the 2020 case, and then the parameter changes for the 2008 case. The parameters are changed to target the different values of interest rates and bank balance sheets in the two cases. The parameter values for the two cases are listed in Table 1.

The length of a period is one quarter. Some parameters are set equal to standard values in the literature: The exponent of the production function is $\alpha=0.35$; The capital depreciation rate is $\delta=0.025$; The relative risk aversion is $\gamma=2$. The value of the capital adjustment cost parameter, $\psi=1$, is also within the range of standard values. The Frisch elasticity of labor supply, $\varphi=0.5$, is close to common econometric estimates. The scale parameters $A$ and $\Phi$ are set so that $y=1$ and $n=1 / 3$, respectively.

I assume that the penalty functions $g$ and $h$ are quadratic ( $\lambda=2$ and $\rho=2$ ), and I show how results change as $\lambda$ and $\rho$ change in the sensitivity section, Section 4.3. Similarly, I set the risk weights to $\omega_{N}=0.25$ and $\omega_{L}=0.5$, and I show how results change as $\omega_{N}$ and $\omega_{L}$ change.

The preference discount factors and friction parameters are set to match the average interest rates in 2016-2019 (after the end of the zero-lower-bound period and before the COVID-19 crisis) and the values of banks' assets and liabilities in 2019. The quarterly interest rates of deposits, Treasury bills, Treasury bonds, and loans are set to $r^{D}=0.0001, r^{M}=0.0033, r^{N}=0.0058$, and $\tilde{r}^{L}=0.0115$, to match the 20162019 averages of the savings rate (FDIC, non-jumbo checking deposit rate, FRED), 3-month Treasury rate (Board of Governors, 3-month Treasury bill rate, FRED), 10year Treasury rate (Board of Governors, 10-Year Treasury constant maturity yield,

FRED), and investment grade bond yield (Moody's, seasoned Baa corporate bond yield, FRED), respectively. The values of banks' assets and liabilities (relative to y) are set to match the corresponding values for all commercial banks in 2019:Q4 (relative to quarterly GDP). Specifically, bank reserves, $R=0.32$, match the cash assets of all commercial banks; banks' holdings of government debt, $B=0.55$, match holdings of Treasury and agency securities; bank loans, $L=1.99$, match bank credit net of Treasury and agency securities; and deposits, $D=2.42$, match the deposits of all commercial banks (Federal Reserve statistical release, Table H.8, Haver Analytics). These values for interest rates and bank balance sheets, the agents' first-order conditions, and the friction definitions (6), (9), and (24) pin down the preference discount factors $(\hat{\beta}, \beta$, and $\tilde{\beta})$ and the friction parameters $\left(A_{g}, A_{h}\right.$, and $\left.A_{z}\right)$.

The sensitivity of the credit risk wedge to output, $\eta_{y}=0.25$, is set to approximate the increase (1 percentage point) in the quarterly credit spread (Moody's Baa corporate bond yield minus 10-Year Treasury yield, FRED) relative to the drop (4 percent) in GDP during the Great Recession. The sensitivity of the credit risk wedge to bank lending, $\eta_{L}$, is especially hard to pin down. I choose the benchmark value, $\eta_{L}=0.042$, so that the yearly credit spread decreases by 8.4 percentage points (equal to the difference between the maximum and median of the corporate bond spread in Table 1 of Flannery et al. 2012) when bank lending decreases by 50 percent. I show how results depend on $\eta_{L}$ in Section 4.3.

The decay parameter for the Treasury bond coupon payments equals $\kappa=1-1 / 40$, so the Treasury bond duration is 10 years. The duration of a Treasury bill is one quarter. I assume that 50 percent of the value of government debt consists of Treasury bills and the remaining 50 percent consists of Treasury bonds, so the model average duration of government debt, 5.125 years, matches the weighted average maturity in the data, approximately (US Treasury Office of Debt Management). Similarly, I assume that both the central bank and private banks hold 50 percent of their government debt holdings in Treasury bills and the remaining 50 percent in Treasury bonds. The first-order autocorrelation coefficients of the policy processes are equal
to $0.9\left(\rho_{Q E}=0.9\right.$ and $\left.\rho_{D L}=0.9\right)$.
Government spending is set to $G=0.15$ to match the ratio of government spending to GDP. The constant $A_{T}$ is set to balance the government budget constraint. The fiscal rule policy coefficient is equal to $\tau=0.01$, so the response of government transfers to government debt is small but sufficient to ensure the existence of an equilibrium.

The calibration for the 2008 case is the same as for the 2020 case, except that parameters are set to match the average interest rates in 2002-2007 and the values of banks' assets and liabilities in 2008:Q3, before the first QE announcement. Specifically, the quarterly interest rates of deposits, Treasury bills, Treasury bonds, and loans are set to $r^{D}=0.0008, r^{M}=0.0068, r^{N}=0.0111$, and $\tilde{r}^{L}=0.0167$, respectively. Banks' reserves, holdings of government debt, loans, and deposits are set to $R=0.09, B=0.31, L=2.04$, and $D=1.87$, respectively.

## 4 Results

In this section, I first examine the effects of QE, comparing 2008 and 2020, and then the effects of direct lending at market and subsidized rates. ${ }^{1}$

### 4.1 Quantitative easing

In the model, QE refers to central bank purchases of Treasury bonds from private banks financed by increased bank reserves. Private banks' portfolios change as a result. Bank reserves, $R_{t+1}$, increase, while banks' holdings of Treasury bonds, $q_{t} N_{t+1}$, decrease by the same amount. Figure 1 shows the aggregate effects of central bank purchases worth 4 percent of annual GDP, the same size as the first announcement of QE in November 2008. The dashed and solid lines refer to the 2008 and 2020 cases, respectively.

[^1]For the 2008 case, the figure shows the increase in bank reserves, $R_{t+1}$, accompanied by the increase in central-bank-held government debt, $\tilde{B}_{t+1}$. Since bank reserves increase, the liquidity premium, $\pi_{t+1}^{l i q}$, decreases. And since banks' holdings of Treasury bonds decrease, banks' risk-weighted assets, $Z_{t+1}$, decrease, and the volatility risk premium, $\pi_{t+1}^{v o l}$, decreases. The decreases in the two premiums lower the loan-deposit spread, $s_{t+1}$, which, in turn, stimulates bank lending, investment, and output. Firms' available funds, one of Fed programs' primary objectives in 2020, increase. Banks expand their deposits to fund the increase in bank loans. Notice that the decrease in the liquidity premium is about 10 times larger than the decrease in the volatility risk premium, so the decrease in the liquidity premium is the main driver of the effects quantitatively.

The credit risk friction amplifies these first-round effects. The output increase lowers the credit risk premium, $\pi_{t+1}^{c r e d}$, further decreasing the loan-deposit spread and increasing investment and output. The predicted total effect on output, 3.1 percent, is between the estimate by Weale and Wieladek (2016) for a QE program worth 4 percent of annual GDP ( $4 \times 0.62$ percent $=2.5$ percent $)$ and the estimate by Baumeister and Benati (2013) for the first round of QE (3.5 percent).

Turning to the 2020 case, the qualitative effects of QE are the same as in the 2008 case. Quantitatively, however, the effects are much smaller. The output increase is 0.5 percent in 2020, 2.6 percentage points smaller than in 2008 . The main cause is that the level of bank reserves (relative to GDP) is 3.5 times greater, so the percent increase in bank reserves is 3.5 times smaller. As a result, the decrease in the liquidity premium, defined by (45), is smaller. Since the decrease in the liquidity premium is the main driver of the effects, the effects of QE are smaller in 2020 than in 2008.

More generally, since bank reserves rose after 2008, this mechanism suggests that later QE programs had smaller effects than the first. For example, if we used the parameter setting of the 2008 case except that we increased bank reserves by the amount announced in November 2008 (4 percent of GDP), the output response would be 2.1 percent, 50 percent smaller than in the 2008 case. In other words, later QE
programs were 50 percent less expansionary because of the increase in bank reserves announced in 2008.

The result that QE was less expansionary in 2020 than in 2008 is similar to the one obtained by Cardamone, Sims, and Wu (2023), but for a different reason. They argue that QE can relax the financial constraints at the bank level but not at the firm level. In 2008, bank lending was limited by financial constraints at the bank level, and QE was effective in relaxing these constraints. In 2020, bank lending was limited by financial constraints at the firm level, and QE was ineffective.

I offer a different explanation. QE works by lowering various interest rate premiums. By injecting liquidity, QE lowers the liquidity premium and the rate of return of illiquid assets. By withdrawing volatility risk, it lowers the volatility risk premium and the rate of return of volatile assets. By stimulating output, it lowers the credit risk premium and firms' borrowing costs. The main driver of the effects is the decrease in the liquidity premium. Since the level of bank reserves was higher in 2020, the effect of a given increase in bank reserves on liquidity and the liquidity premium was smaller, and the overall economic impact was less expansionary.

The two arguments can work together to indicate that QE was less expansionary in 2020 than in 2008. One way to incorporate their mechanism in my model would be to assume that firms' borrowing from the private sector was unconstrained in 2008 ( $\eta_{L}=$ $0)$ and more constrained than in the baseline in $2020\left(\eta_{L}=0.1\right)$. This assumption would lead to more expansionary predicted effects in 2008 (with output increasing by 3.5 percent, 0.4 percentage points more than in the baseline) and less expansionary predicted effects in 2020 (with output increasing by 0.4 percent, 0.1 percentage points less than in the baseline). In other words, the output response to QE would be 0.5 percentage points smaller in 2020 than in 2008 because of their mechanism, modeled as an increase in $\eta_{L}$ from 0 to 0.1 . Such a decrease in output response is sizeable but much smaller than the one due to my mechanism, 2.6 percentage points.

### 4.2 Direct lending to firms

In a direct lending program, the central bank lends to firms and finances the loans with increased bank reserves. Bank reserves, $R_{t+1}$, and firms' borrowing from the central bank, $\tilde{L}_{t+1}$, increase by the same amount. Figure 2 shows the effects of direct lending in 2008 (dashed line) and 2020 (solid line). The loans are worth 4 percent of annual GDP, the same size as the QE program we considered.

Some mechanisms through which direct lending works are similar to those of QE. Since bank reserves increase, the liquidity premium, $\pi_{t+1}^{l i q}$, decreases. Firms substitute central bank loans for private bank loans. As bank lending, $L_{t+1}$, decreases, banks' risk-weighted assets, $Z_{t+1}$, decrease, and the volatility risk premium, $\pi_{t+1}^{v o l}$, decreases. The decreases in the two premiums lower the loan-deposit spread, $s_{t+1}$, and stimulate investment and output. The output increase lowers the credit risk premium, $\pi_{t+1}^{c r e d}$, further decreasing the loan-deposit spread and amplifying the first-round effects.

Like QE, central bank lending was less expansionary in 2020 than in 2008. A direct lending program worth 4 percent of GDP would have raised real GDP by 3.4 percent in 2008 and 0.8 percent in 2020. The reason behind the diminished stimulus is the same for direct lending as for QE. The main driver of the effects is the decrease in the liquidity premium. Since bank reserves were greater in 2020, the decreases in the liquidity premium and the loan-deposit spread were smaller.

Direct lending, however, works through two additional channels that make it more expansionary than QE. First, QE decreases banks' holdings of Treasury bonds, while direct lending decreases bank loans. Since bank loans carry more volatility risk than Treasury bonds, direct lending withdraws more risk from the private sector and lowers the volatility risk premium more than QE. In the model, the loan risk weight, $\omega_{L}$, is greater than the bond risk weight, $\omega_{N}$, so direct lending lowers banks' risk-weighted assets, the volatility risk premium, and the loan-deposit spread more than QE. This channel, however, is quantitatively small, as can be inferred from the small response of the volatility risk premium.

Second, while QE stimulates bank lending, direct lending substitutes it. Since
firm-level financial frictions worsen with bank lending ( $\eta_{L}>0$ ) but not direct lending, direct lending lowers the credit risk premium more than QE. This second channel is more sizeable, as evident from the response of the credit risk premium.

The second channel is the one Cardamone, Sims, and Wu (2023) propose to derive that direct lending was more expansionary than QE in 2020. They assume that firms' borrowing from the private sector was constrained by firms' cash flow in 2020. Since QE works by stimulating firms' borrowing from the private sector, its expansionary effects were constrained. In contrast, the effects of central bank direct lending were unconstrained. On the one hand, my model confirms that their channel is the main one through which direct lending is more expansionary than QE. On the other hand, the channel size is much smaller than shown in their paper, where the crucial parameter, $\varphi$, controls the cash-flow constraint and is not calibrated but set to illustrate the main message. In 2020, a direct lending program worth 4 percent of GDP would have raised output by 0.3 percentage points more than QE according to my model (compare this paper's Figures 1 and 2), while it would have raised output by 6 percentage points more according to theirs (compare their paper's Figures 3 and 4 and multiply their output responses by 4 since the size of their lending shock is 1 percent of GDP).

Summing up all the effects, my model indicates that direct lending is more expansionary than QE. In 2008, a direct lending program worth 4 percent of GDP and a QE program of the same size would have expanded output by 3.4 percent and 3.1 percent, respectively (Figures 1 and 2). In 2020, they would have expanded output by 0.8 percent and 0.5 percent, respectively.

As both QE and direct lending became less expansionary, direct lending became relatively more expansionary than QE. In the initial period, the expansionary effect of direct lending would have been 10 percent greater than QE in 2008, while it would have been 60 percent greater in 2020. In other words, to achieve the same initial expansionary effect of a lending program, the Federal Reserve would have needed a 10 percent larger QE program in 2008 and a 60 percent larger QE program in 2020.

### 4.2.1 Subsidized direct lending

Direct lending can have more expansionary effects if the central bank provides the loans at a subsidized rate lower than the market rate. In this case, there are two additional effects.

First, the loans provide a lump-sum subsidy to firms equal to the size of the loan times the difference between the market rate and the subsidized rate. Quantitatively, this effect is tiny (not shown). The size of the effect on firms' available funds, investment, and output is two orders of magnitude smaller than the size of unsubsidized direct lending.

Second, the loans can lower firms' marginal borrowing costs and user cost of capital. This effect depends on how much the central bank manages to lower firms' marginal borrowing rates. On the one hand, subsidized direct lending may not affect firms' marginal borrowing rates if firms borrow from banks rather than the central bank on the margin. On the other hand, a lending facility can lower firms' marginal borrowing rates by serving as a backstop even if the facility is not used, as shown by the easing of financial conditions that followed the announcement of the Federal Reserve's Corporate Credit Facilities in March 2020 (Boyarchenko, Kovner, and Shachar 2020, and Clarida, Duygan-Bump, and Scotti 2021).

Figure 3 shows the overall effect of subsidized direct loans worth 4 percent of GDP in the case where the loans lower firms' marginal borrowing rate by 1 percentage point. In the initial period, output increases by 3.7 percent and 1.1 percent in 2008 and 2020, respectively. In both years, the output response is 0.3 percentage points larger than in the case of unsubsidized direct lending. The additional expansionary effect of subsidized direct lending would be proportionally larger if firms' marginal borrowing rate dropped more.

### 4.3 Sensitivity analysis

One message of this paper is that the expansionary impact of Fed programs is inversely related to the level of bank reserves before the program introduction. This relationship depends on the parameters of the liquidity friction, $A_{g}$ and $\lambda$. Another message is that QE is less expansionary than direct lending because it worsens the financial frictions that constrain firms' borrowing from the private sector, while direct lending mitigates them. This result depends on $\eta_{L}$, the parameter that controls the credit risk friction associated with firms' borrowing from banks. The other parameters change some results quantitatively but do not affect the two main messages.

Figure 4 plots the response of output to QE and unsubsidized direct lending in 2020 for different values of the key parameters.

The parameters of the liquidity friction, $A_{g}$ and $\lambda$, are important for the results because changes in the liquidity premium are the main drivers of the economy's response to Fed programs. A greater scale parameter, $A_{g}$, implies a greater liquidity premium, $\pi^{l i q}$, a wider loan-deposit spread, $s$, and larger effects of Fed programs on the premium, the spread, and output. If $A_{g}$ doubles, the liquidity premium doubles, the loan-deposit spread widens by 32 basis points, the output response to QE doubles in the initial period, and the output response to direct lending increases by 50 percent (first row of Figure 4).

The sensitivity to $A_{g}$ also sheds light on how results would change if the liquidity friction worsened during crises. Banks likely raise their target reserve-deposit ratio during crises partly to meet more volatile and uncertain households' withdrawals and deposits. A higher target ratio could be captured in the model by setting a greater value of $A_{g}$, implying a greater liquidity friction, a greater penalty for holding a low level of reserves, and a higher reserve-deposit ratio in the steady state. Then, the model sensitivity to $A_{g}$ indicates that Fed programs would be more expansionary in response to crises that worsened financial frictions and raised the liquidity premium and loan-deposit spread.

The output response is also sensitive to the exponent, $\lambda$, of the penalty function
associated with the liquidity friction. Intuitively, the higher $\lambda$, the larger the response of the liquidity friction and output to Fed programs that change the deposit-reserve ratio. In the initial period, the output response to both Fed programs is more than 1 percentage point larger in the case where $\lambda=10$ than $\lambda=2$ (second row of Figure 4 ).

In contrast, model results are not that sensitive to the parameters of the volatility risk friction, $A_{h}$ and $\rho$, since changes in the volatility risk premium are small and play a small role in the economy's response. In the initial period, the output response does not depend on $\rho$ (third row of Figure 4) or $A_{h}$ (not shown).

In theory, with a higher loan risk weight, $\omega_{L}$, direct lending withdraws more risk from the private sector and is more expansionary. However, the change in the output response is tiny (fourth row of Figure 4). Similarly, model results do not depend significantly on the bond risk weight, $\omega_{N}$ (not shown).

Turning to the credit risk friction, the sensitivity to $\eta_{y}$ is small, reflecting the limited role that the credit risk friction and the credit risk premium play (fifth row of Figure 4). Model results do not depend significantly on the level of the credit risk wedge, $A_{z}$ (not shown).

Model results depend on $\eta_{L}$ intuitively (sixth row of Figure 4). With a higher $\eta_{L}$, changes in bank lending have larger effects on the credit risk friction. QE is less expansionary, since it stimulates bank lending and worsens the credit risk friction by more. In contrast, direct lending is more expansionary since it substitutes bank lending and mitigates the friction by more. However, even if $\eta_{L}=1$, more than double its calibrated benchmark value, the financial friction remains much smaller than in Cardamone, Sims, and $\mathrm{Wu}(2023) .{ }^{2}$ With $\eta_{L}=1$, direct lending would have raised output by 0.5 percentage points more than QE, according to my model. With lower values of $\eta_{L}, \mathrm{QE}$ and direct lending have more similar effects. If $\eta_{L}=0$, the effects

[^2]of QE and direct lending are approximately the same. The effects of QE and direct lending on output would be identical if the risk weights of bonds and loans were the same $\left(\omega_{N}=\omega_{L}\right)$ and the risk wedge did not depend on bank lending $\left(\eta_{L}=0\right)$.

The sensitivity to the more standard parameters is in line with other calibrated dynamic general equilibrium models. For instance, one parameter value important for the results is the Frisch elasticity of labor supply, $\varphi$. The output response depends on $\varphi$ intuitively. Larger values of the Frisch elasticity of labor supply lead to larger effects of Fed programs on labor, resulting in larger effects on output (seventh row of Figure 4).

## 5 Conclusion

In the model I have presented, QE and direct lending to firms work through three channels. Expanding bank reserves lowers the liquidity premium that non-reserve assets earn above the return of on-demand deposits. Decreasing the net supply of assets with volatile returns, such as Treasury bonds and bank loans, lowers the volatility risk premium. Stimulating the economy lowers the credit risk premium. All these channels lower the loan-deposit spread and stimulate firms' investment and output.

The model indicates that the liquidity premium channel is, quantitatively, the most important. Since bank reserves were greater in 2020 than in 2008, the liquidity premium channel was weaker, and Fed programs were less expansionary. More generally, since bank reserves rose after 2008, this mechanism suggests that later QE programs had smaller effects than the first.

Direct lending to firms is more expansionary than QE because QE stimulates bank lending and worsens the credit risk frictions associated with firms' borrowing from the private sector, while central bank direct lending substitutes bank lending and mitigates the frictions. This result confirms the argument of Cardamone, Sims, and Wu (2023) that direct lending was more expansionary than QE in 2020 because firmlevel financial frictions constrained bank lending to firms and made QE less effective.

According to the model, a QE program worth 4 percent of GDP would have raised real GDP by 3.1 and 0.5 percent in 2008 and 2020, respectively. A direct lending program of the same size would have raised real GDP by 3.4 and 0.8 percent in 2008 and 2020, respectively. As bank reserves increased, QE and direct lending became less expansionary over time, but direct lending became relatively more expansionary than QE. To achieve the same expansionary effect of a lending program, the Federal Reserve would have needed a 10 percent larger QE program in 2008 and a 60 percent larger QE program in 2020. For given costs, risks, and constraints on the use of QE and direct lending, the relative increase in the stimulus provided by direct lending may have been one reason why the Fed resorted to it in 2020 but not in 2008 and suggests that the Fed may use it again in the next crisis.

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|  | Description | 2008 Case | 2020 case | Targeted moments |
| :---: | :---: | :---: | :---: | :---: |
| $\alpha$ | production function exponent | 0.35 | 0.35 |  |
| $\delta$ | capital depreciation rate | 0.025 | 0.025 |  |
| $\psi$ | capital adjustment cost | 1 | 1 |  |
| $\gamma$ | relative risk aversion | 2 | 2 |  |
| $\varphi$ | Frisch elasticity of labor supply | 0.5 | 0.5 |  |
| A | production function scale | 0.9700 | 0.9258 | $y=1$ |
| $\Phi$ | labor disutility scale | 22.55 | 22.99 | $n=1 / 3$ |
| $\hat{\beta}$ | households' pref. discount factor | 0.9992 | 0.9999 | $r^{D}$ |
| $\beta$ | banks' pref. discount factor | 0.9625 | 0.9720 | $r^{M}$ and $Z / V$ |
| $\tilde{\beta}$ | firms' pref. discount factor | 0.9836 | 0.9886 | $\tilde{r}^{L}$ |
| $A_{g}$ | liquidity friction scale | 0.0112 | 0.0078 | $r^{D}, r^{M}$, and $D / R$ |
| $\lambda$ | liquidity friction exponent | 2 | 2 |  |
| $A_{h}$ | risk friction scale | 0.0184 | 0.0110 | $r^{M}, r^{N}$, and $Z / V$ |
| $\rho$ | risk friction exponent | 2 | 2 |  |
| $\omega_{N}$ | risk weight of Treasury bonds | 0.25 | 0.25 |  |
| $\omega_{L}$ | risk weight of loans | 0.5 | 0.5 |  |
| $A_{z}$ | steady-state credit risk wedge | 0.0013 | 0.0032 | $r^{N}, \tilde{r}^{L}$, and $Z / V$ |
| $\eta_{y}$ | sensitivity of $z$ to $y$ | 0.25 | 0.25 |  |
| $\eta_{L}$ | sensitivity of $z$ to $L$ | 0.042 | 0.042 |  |
| $\kappa$ | bond coupon decay | 0.975 | 0.975 | 10-year bond duration |
| $q N / B$ | Treasury bond share of govt debt | 0.5 | 0.5 | govt debt duration |
| $\rho_{Q E}$ | quantitative easing autocorrelation | 0.9 | 0.9 |  |
| $\rho_{D L}$ | direct lending autocorrelation | 0.9 | 0.9 |  |
| $G$ | government spending | 0.15 | 0.15 |  |
| $\tau$ | response of govt transfers to debt | 0.01 | 0.01 |  |
| $A_{T}$ | steady-state govt transfers | -0.142 | -0.146 | govt budget balance |
| $R$ | bank reserves | 0.09 | 0.32 |  |
| $B$ | govt debt held by banks | 0.31 | 0.55 |  |
| $L$ | bank loans | 2.04 | 1.99 |  |
| $D$ | bank deposits | 1.87 | 2.42 |  |

Table 1: Parameters and steady-state values. Note: The length of a period is 1 quarter.


Figure 1: Effect of quantitative easing. Treasury bond purchases are worth 4 percent of annual GDP, with a 0.9 first-order autocorrelation. Note: The dashed and solid lines refer to the 2008 and 2020 economies, respectively. The first four subplots are expressed in percentage points; the others are in percent.


Figure 2: Effect of direct lending. Loans are worth 4 percent of annual GDP, with a 0.9 first-order autocorrelation. Note: The dashed and solid lines refer to the 2008 and 2020 economies, respectively. The first four subplots are expressed in percentage points; the others are in percent.


Figure 3: Effect of subsidized direct lending. Loans are worth 4 percent of annual GDP, with a 0.9 first-order autocorrelation. The subsidized rate is 1 percentage point below the market rate. Firms' marginal borrowing rate becomes equal to the subsidized rate. Note: The dashed and solid lines refer to the 2008 and 2020 economies, respectively. The first four subplots are expressed in percentage points; the others are in percent.

Effect of quantitative easing on $y_{t}$







Figure 4: Parameter sensitivity. Effect of quantitative easing and direct lending on output. Note: Both the dashed and solid lines refer to the 2020 economy. Parameters are set at their benchmark values except for the parameters in the legend. The output response is expressed in percent.


[^0]:    ${ }^{\dagger}$ Department of Economics, Finance, and Quantitative Analysis, Coles College of Business, Kennesaw State University, 560 Parliament Garden Way, Kennesaw GA 30144. E-mail: focchino@kennesaw.edu.

[^1]:    ${ }^{1}$ The model is solved using the Dynare software (first-order linear approximation and Klein's QZ decomposition solution method).

[^2]:    ${ }^{2}$ I view $\eta_{L}=0.042$ as the most plausible value, as explained in the calibration section. The largest value that I consider, $\eta_{L}=0.1$, implies that a firm's annual (/quarterly) credit risk spread would decrease by forty (/ten) percentage points if the firm managed to repay all its bank loans (a decrease in bank loans by 100 percent). I view this parameter value as too high since average speculativegrade corporate bond spreads are well below 40 percentage points, so they cannot decrease by 40 percentage points.

