Tullock contest alliances with proportional prize-sharing agreements: Private collective action mechanisms?

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Abstract: This paper focuses on humanity's supposedly irrational behavior in conflict decision-making, challenging two rationalist puzzles in political science and economics: war's inefficiency and alliance formation. More specifically, we ask whether alliances can benefit the allies. Standard Tullock contest alliances are plagued by free riding, undermining successful collective action. In a three-party contest environment when two of the parties ally, input substitution and fixed prize division hinder collective action. Analyzing the same contest with input-cost complementarity and proportional prize division, we propose a transformative solution that avoids the usual problems of alliance formation and stability. While input-cost complementarity partially mitigates those concerns, a proportional prize-sharing agreement offers a comprehensive remedy, ensuring equitable contributions and gains for the allied parties. The proposed approach not only resolves the alliance-formation puzzle but also enhances the allies' prospects for success.

Keywords: Contests, resource conflict, alliance formation puzzle, collective action, NATO

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1. Introduction

Under the guidance of Vernon Smith, humanomics has expanded our ideas about the boundaries of economic inquiry and its methods. In his book on the subject, coauthored with Bart Wilson, humanomics is described as, "the study of the very *human* problem of simultaneously living in these two worlds, the personal social and the impersonal economic" (Smith & Wilson, 2019: 2; emphasis in the original). The present work examines one manifestation of that balancing act, between the more familiar relationships forged between friends or allies and the less familiar, more competitive relationships with strangers. Specifically, we examine how considerations of fairness among allies can alter conflict scenarios with non-allied parties. In doing so, we address contributions to the humanities, such as political science, and the traditionally more humanistic-leaning social sciences literature, which assert and even demonstrate that decision-making in conflict settings is irrational. In particular, that literature has given rise to *war's inefficiency puzzle* (Fearon, 1995), which questions why parties would ever engage in wasteful conflict when peaceful settlement would lead to superior outcomes for everyone, as well as other critiques of the rationalist paradigm.[1] In economics, the *alliance-formation puzzle* has raised similar questions about the logic of collective action in conflict scenarios because allied parties in models of conflict often end up worse off, in payoff terms, than if they had fought an enemy independently.

With those puzzles in mind, we consider the tragic and seemingly self-defeating aspect of humanity known as conflict; specifically, the kind that may be escalated by alliance formation. We do so through the lens of economics by considering a contest model, in which conflicting parties choose levels of inputs, which may represent spending on armaments or other conflict-related (costly) efforts. The winner of the contest then depends on the contest success function, which maps parties' input

[1] See Blattman (2022) for a recent overview of the war inefficiency puzzle.

contributions to their odds of victory. The appeal of an alliance is that allied members can pool their contest inputs, thereby increasing those odds.

A crucial aspect of alliances in contest models—and in real life alliances such as NATO, as we explain in the next section—is the division rule that specifies how allies will split or share the prize should they win. Here we explore the rule of *proportional* prize division, whereby allies divide the total prize based on their relative input contributions. In doing so, we find that a shared social norm of fairness can rationalize an alliance's participation in a conflict, addressing the alliance formation puzzle.

It is well-established that standard Tullock (1980) contest alliances in conflict or rent-seeking settings elicit free-riding (e.g. Ke, Konrad, & Morath, 2013). When the allied parties pool their independent contest inputs together, the combined spending could increase their odds of victory, but no individual party wants to carry the burden for the group. In three-party contests, for example, when two of the parties become allies, that perverse effect is sufficiently strong to *lower* the alliance's likelihood of victory and thwart expected payoff gains within the alliance as compared to a standard three-party contest without alliances. In fact, the allies' input choices are not even uniquely determined with prize sharing because the model provides a solution only for their combined input spending in equilibrium. Perhaps most unexpectedly, the existence of the alliance can generate payoff gains for the third, *unallied* party.

Under the standard contest treatment, the collective action problem arises because allies rely on one another's inputs as substitutes in the contest success function, and because they exogenously agree (ex ante) to a fixed split of the prize regardless of their input contributions in the event of winning. In contrast, we consider a two-party alliance in a three-party contest that features input *cost complementarity* (though inputs remain perfect substitutes in the allies' contest success function) as well as an endogenous proportional prize division agreement based on the allies' individual input contributions. Even with the same Tullock-style contest success function featured in standard alliance models, input cost complementarity provides part of the answer to the alliance formation puzzle: the payoffs to the allies rise relative to the baseline case of a contest without alliances due to the cost synergies. Input complementarity alone does not fully correct the collective action problem for the alliance, however. A proportional prize division agreement based on input contributions is the component of the model that fully resolves the collective action problem, presenting a solution to the alliance formation puzzle. With both input cost complementarity and proportional prize division, allies' input allocations are determined uniquely; moreover, the alliance's probability of winning increases relative to the situation without alliances, as do the payoffs to each ally.

In the next section we review the theoretical and empirical literature on alliances to motivate our approach. While the literature on conflict-alliance formation generally identifies sub-optimal decision-making, none of the extant studies considers how an alliance might accommodate a shared understanding of social norms (such as fairness) to facilitate institutional solutions to the alliance formation puzzle. In the third section, we present our contest model of conflict, first without alliance formation to provide a baseline of comparison, then with alliance formation under three different sets of assumptions regarding how the allies' efforts combine to determine their share of the prize if victorious. Just as the role of informal constitutions in various economic settings has been found to be important in improving social outcomes (see, e.g., Leeson and Skarbek, 2010; Leeson, 2011), we show that such institutions based on a mutual understanding of fairness can play a key role in humanizing conflict situations. We then conclude with a discussion of how our approach and results may apply to real-world alliance situations, in particular NATO.

2. Previous literature and motivation

A full summary of the relevant theoretical models is provided by Matthews and Sanders (2019). Most of those studies establish conditions for forming alliances in the context of Tullock's (1980) contest success functions, focusing on coordinated input decisions by allies and comparing the expected rents accruing to contesting parties with and without alliance formation.[2] For example, influenced by public-choice reasoning, Skaperdas (1998), Konrad (2012), Ke, Konrad, and Morath (2013), Boudreau, Rentschler, and Sanders (2019), and Boudreau, Sanders, and Shunda (2019) have probed various aspects of alliance formation in theoretical models of conflict.

Previous papers examining conflict in contest-model settings generally do not find conflict to be an efficient or even rational outcome (see, e.g., Mitchell 2019; Munger 2019; or Chang, Potter, & Sanders, 2007). As a central example, Fearon (1995) specifies a Tullock (1980) contest model and shows that the conflict outcome always is inferior to a bargaining solution such that the observation of conflict represents a paradox. That paradox spawned a vast literature on war's inefficiency, which asks the obvious question: if war is so inefficient, why is it so commonplace in human history? It likewise is well-established that standard Tullock (1980) contest alliances (e.g., in conflict or rent-seeking settings) elicit freeriding. In turn, the effects of free-riding have been shown to give rise to an alliance formation puzzle in three-party Tullock contests (Konrad & Kovenock, 2009; Ke, Konrad, & Morath, 2013; Boudreau, Rentschler, & Sanders, 2019; Boudreau, Sanders, & Shunda, 2019; Mathews & Sanders, 2019; Konishi & Pan 2020): smaller input contributions by allied parties end up lowering the alliance's equilibrium probability of victory and the allies' payoffs relative to un-allied conflict.

Solutions to the alliance formation puzzle other than those we summarize here have been proposed. They include capacity constraints (Konrad & Kovenock, 2009; Ehrlich, Harmon, & Sanders, 2020), input complementarity in the contest success function (Boudreau, Rentschler, & Sanders, 2019), and sufficiently noisy contests in which the collective action problem does not lower expected ally payoffs (Boudreau,

[2] One notable exception is Konrad (2012), who considers budget-constrained parties in a contest model who have the option of revealing private budget information to their alliance partners.

Sanders, & Shunda, 2019). Herein, we consider the role of allied prize-division agreements in solving the alliance formation puzzle. While other solutions to the alliance formation puzzle have been presented, no previous contribution to the literature has studied the possibility that the parties to contest can address the within-contest alliance free-riding problem. We conjecture that allies can solve the alliance formation puzzle in spite of a persisting collective action problem.

In what follows, we consider the roles of cost complementarity and proportional prize division in jointly addressing the alliance formation puzzle and moderating within-alliance free riding. Proportional prize division is a mechanism by which allies divide the prize endogenously in the event of allied victory. Specifically, parties divide the prize according to the relative input contributions of each allied party. Such a sharing rule may be formally negotiated by treaty (an alliance constitution) and enforced by an overseeing (international) organization, or informally by a mutual understanding of the forward value of the alliance beyond a one-shot conflict or contest. Similarly, an outside superpower that is aligned with both allied parties can enforce a prize-sharing agreement in the interest of maintaining the strength of a larger treaty organization.

Our focus on the roles of complementarity and proportional division is motivated by the experience of perhaps the most prominent (current) military alliance, NATO, over its more than 70-year existence. The early years of NATO, from roughly 1949 to 1966, were defined by an emphasis on nuclear deterrence, as chronicled by Sandler and Murdoch (2000). The non-rival and non-excludable nature of nuclear deterrence meant that during that period, allied inputs were highly substitutable, leading to the *exploitation hypothesis* of Olson and Zeckhauser (1966). They predicted that larger, wealthier economies would carry the defense burden for smaller countries, and the hypothesis was at the time borne out. Multiple studies (e.g., Sandler & Forbes, 1980; Khanna & Sandler, 1996) find significant, positive rank correlations between NATO countries' overall sizes in terms of gross domestic product (GDP) and their military expenditures relative to GDP (ME/GDP) for the years 1960 to 1966.[3] Oneal and Elrod (1989), meanwhile, find that a significant percentage of the variation in NATO countries' ME-to-GDP ratios was explained by GDP for the years 1953-1968. For that era, the evidence favors the notion that smaller allies free ride on their larger counterparts when defense inputs are substitutable. After that era, however, things changed.

Following the Cuban missile crisis in 1967, NATO adopted directive MC 14/3, which began what Sandler and Murdoch (2000) refer to as the flexible response era, running roughly from 1967 to 1990. Rather than a doctrine of mutually assured destruction, directive MC 14/3 "permitted NATO to respond in a measured fashion to Warsaw Pact aggression. With this doctrine, a small conventional force incursion, initiated by the Warsaw Pact, would be met with commensurate conventional countermeasures" (Kim & Sandler, 2020: 403). In other words, rather than immediate escalation, the allies agreed to rely on smaller-scale, more targeted approaches, leaving nuclear weapons as a measure of last resort. The MC 14/3 directive meant that defense inputs became more complementary and alliance benefits became more ally-specific (Murdoch and Sandler, 1984).

During the flexible response era, when NATO spending efforts were more complementary, evidence of allies' free riding disappears. Sandler and Forbes (1980) found no significant correlation between GDP and ME/GDP for NATO countries over the 1967-1975 period, while Khanna and Sandler (1996) found no significant such correlations for 1967-1992. Similarly, for 1969-1984, Oneal and Elrod report that only an insignificant percentage of the variance in NATO countries' ME-to-GDP ratios was explained by their GDPs. The larger, wealthier alliance members no longer were bearing disproportionate shares of total military spending. Additionally, however,

[3] Olson and Zeckhauser also found a significant correlation between NATO allies' size as measured by gross national product (GNP) and their ME/GNPs for the year 1964 (only), supporting their original hypothesis.

Sandler and Forbes (1980) constructed a measure of ally benefits by equally weighting each ally's share of NATO's GDP, its share of NATO's total population, and its share of NATO's exposed borders. They then compared the distribution of their benefit measure to the distribution of the ally's individual ME to total NATO ME, finding a closer benefit-burden match in 1975 than in 1960. Extending that work, Khanna and Sandler (1996) found no significant difference between the benefit and burden distributions at five-year intervals for 1965, 1970, 1975, 1980, and 1990,[4] meaning that both spending burdens and benefits were shared more proportionally throughout the period.

Evidence of the importance of proportional benefit sharing among NATO allies has only grown stronger in recent years. While Sandler and Murdoch (2000) reported evidence of close benefit-burden matching for every year during the 1990s, Sandler and Shimizu (2014) did not find any such evidence for the years 2002-2009, nor did Kim and Sandler (2020) for 2011-2017. This matters because Sandler and Shimizu (2014) found increasing evidence of free riding starting in 2004 as reflected by the correlation between allies' GDP and ME/GDP measures. Kim and Sandler (2020) then found significant, positive correlations between the two for every year from 2011 to 2017, and updates by George and Sandler (2022) and Kim and Sandler (2024) have continued to show evidence of free riding during the 2000s, particularly 2010-2020. Thus, the reappearance of a disconnect between burdens and benefits (disproportionate prize sharing) was followed by the reappearance of free-riding behavior, motivating our emphasis on proportional division.

The specific role of complementary costs for military allies has been noted by Sandler (1999), who emphasized the role of shared borders resulting in lower patroling costs, particularly in conjunction with NATO expansion. Another example can be found in the US presence in the Syrian civil war, where air support helped coordinate ground attacks and reduce the human toll or cost of ground fighting. Conversely, boots on the

[4] The significant difference between benefits and burdens around 1985 has been attributed to the Reagan-era military buildup reaching its peak around then (Sandler & Murdoch, 2000).

ground in Syria helped identify anti-aircraft defense positions and air targets, thus lowering the overall cost of air attacks. Such complementarities are analyzed in Oyewole (2015) and Pirnie et al. (2005). Pirnie et al. (2005: 14) state in a RAND consultancy report to the US military on the First Gulf War that

> In the first perspective, counter-land air power provides additional fire to supplement those of friendly land forces in the close battle. It contributes to victory in the decisive close battle.... From this perspective, the effects of aerial fires are not fundamentally different from those of land-based fires, particularly artillery fire, and the relationship between the two is one of fairly straightforward substitution. Aerial firepower offers particular advantages relative to artillery and rockets, but terrestrial firepower could compensate for its absence. Substitution may be intermittent, with ground forces calling on air power to fill temporary firepower shortfalls during intense combat or other emergencies, or long-term. The former is well illustrated by the use of air power to help destroy frontline Iraqi units in southern Kuwait and Iraq prior to the start of the coalition ground offensive in Operation Desert Storm, when thousands of attack sorties were added to the weight of a massive artillery bombardment to clear the way through the Iraqi defenses.

Going beyond military examples, Elmuti, Abebe, and Nicolosi (2005) report evidence of cost reductions flowing from alliances between private firms and universities in the world of R&D contests (e.g., the Massachusetts Institute Technology has partnered with Ford Motor Company since 1997). Kang and Sakai (2000) chronicle the substantial number of cross-border strategic alliances between firms from a variety of industries, including pharmaceuticals, automobiles, and information and communication technology, in part owing to multiplying research costs and shorter product life cycles. As in the case of NATO, however, the way such cost savings are shared is crucial to an alliance's success. In a quantitative study of international business alliances, Arino and Ring (2010: 1054) conclude,

> We find that perceptions of fairness types shape the partners' decision-making logics (a property rights logic, a control rights logic, and a relational quality logic), which in turn influence the partners' evaluations of efficiency and equity of the proposed alliance and their decision on whether or not to form it.

With those features in mind in the next section, we model a two-party alliance in a three-party contest that features an input cost complementarity technology and an enforceable proportional prize-sharing agreement motivated by fairness. We find that input cost complementarity causes the alliance formation puzzle to disappear conditionally by generating allied cost efficiencies but does not fully overcome the allies' collective action problem. However, a proportional prize division agreement based on individual input contributions does fully resolve that problem. Taken together, those two measures unconditionally raise the expected payoff to allied parties, thus solving the alliance formation puzzle.

3 Theory: Three-party contest models with and without alliance

3.1 Contest timing and basics

We consider a contest game between three players, $\{i, j, k\}$. All three parties simultaneously choose input contributions $g_z \geq 0, z \in \{i, j, k\}$, in the pursuit of a commonly valued prize representing some valuable resource. For simplicity, we normalize the value of the prize to unity. The contest success function mapping those input decisions into probabilities of victory takes a Tullock-style or "ratio" form, but the specification depends on whether an alliance is forged.

When allies are allowed, parties j and k form the alliance—perhaps based on a treaty arrangement, or perhaps based on a mutual understanding of the conflict setting—

against party i. As in the literature, the alliance allows its members to engage in resource pooling; the difference in our treatment is that pooling involves complementarities. Inputs are privately costly for all parties, but if j and k ally, their resources raise the probability of victory for the alliance as a whole. In the event of winning the contest, the alliance immediately shares the prize according to a pre-negotiated treaty or constitutional rule. The value of the prize and the allies' independent input contributions are common knowledge such that the game is complete information in nature.

3.2 The no-alliance case

We first consider a standard, symmetric, three-party contest without alliances to serve as a benchmark. In our first case, the contest success function takes the standard Tullock (1980) form, wherein the probability of victory for party $z \in \{i, j, k\}$, p_z, is given by

$$p_z = \frac{g_z}{g_i + g_j + g_k}.$$

We denote the unit cost of the input contributions of party $z \in \{i, j, k\}$ as $\frac{1}{f_z^\alpha}$, where $f_z > 0$ represents party z's idiosyncratic cost parameter and $\alpha \in [0,1)$ is a common parameter that will determine the extent of cost complementarity in subsequent cases.[5]

Each party chooses its (irrevocable) input allocation, g_z, to maximize the following objective function:

$$\max_{g_z} \pi_i = \frac{g_z}{g_i + g_j + g_k} \cdot 1 - \frac{1}{f_z^\alpha} g_z.$$

[5] We restrict α to a range that allows for interior, pure-strategy equilibria in all four versions of the model considered here.

We assume that $f_i = f_j = f_k = f$ to save on notation. From the three parties' objective functions we obtain the following three first-order conditions:

$$\frac{g_j + g_k}{(g_i + g_j + g_k)^2} = \frac{1}{f^\alpha} \tag{1}$$

$$\frac{g_i + g_k}{(g_i + g_j + g_k)^2} = \frac{1}{f^\alpha} \tag{2}$$

$$\frac{g_i + g_j}{(g_i + g_j + g_k)^2} = \frac{1}{f^\alpha} \tag{3}$$

Given the symmetry of the first-order conditions, we obtain the best-response function $g_i = g_j = g_k$ and substitute back into the first-order conditions (1)-(3). Solving accordingly, we find the symmetric equilibrium input choices, $g_i^{NA} = g_j^{NA} = g_k^{NA} = \frac{2}{9} f^\alpha$, probabilities of victory, $p_i^{NA} = p_j^{NA} = p_k^{NA} = \frac{1}{3}$, and payoffs, $\pi_i^{NA} = \pi_j^{NA} = \pi_k^{NA} = \frac{1}{9}$, where the superscript NA indicates that the solutions are for the "no-alliance" case. Total rent dissipation, $g_i^{NA} + g_j^{NA} + g_k^{NA}$, therefore is equal to six-ninths of the value of the contest's total prize leaving three-ninths of the prize in net economic rents for the contestants, $\pi_i^{NA} + \pi_j^{NA} + \pi_k^{NA}$.[6]

3.3 Standard alliance with exogenous prize division

We now consider a standard two-party alliance in a three-party contest setting. The alliance is "standard" in the sense that the allies' inputs enter the contest success function as perfect substitutes; they divide the prize exogenously and evenly in the event of an allied victory (see, e.g., Ke, Konrad, & Morath, 2013). We assume that an alliance

[6] The imperfectly discriminating nature of the Tullock (1980) contest prevents the full value of the prize from being dissipated, unlike perfectly discriminating mechanisms such as the all-pay auction.

between j and k is formed with no transaction costs (perhaps arising from a shared history). In our second treatment, parties choose their inputs $\{g_i, g_j, g_k\}$ to maximize

$$\max_{g_i} \pi_i = \frac{g_i}{g_i+g_j+g_k} \cdot 1 - \frac{1}{f_i^{\alpha}} g_i \tag{4}$$

$$\max_{g_j} \pi_j = \frac{g_j+g_k}{g_i+g_j+g_k} \cdot \frac{1}{2} - \frac{1}{f_j^{\alpha}} g_j \tag{5}$$

$$\max_{g_k} \pi_k = \frac{g_j+g_k}{g_i+g_j+g_k} \cdot \frac{1}{2} - \frac{1}{f_k^{\alpha}} g_k \tag{6}$$

where the perfect substitutability of the allies' inputs is reflected in their shared probability of victory, $p_j = p_k \equiv p_{jk} = \frac{g_j+g_k}{g_i+g_j+g_k}$; the unallied party i's probability of victory is $p_i = 1 - p_{jk}$, which is the same as in the case of no alliances. The allies also receive a fixed half of the prize in the event of winning, in accordance with an exogenous, fixed rule of equally sharing the spoils of victory.

We again let $f_i = f_j = f_k = f$ and solve for the following known equilibrium allocations by following the same algorithm as in process as in Section 2.2. Taking first-order conditions and solving, we have that $g_i^{SA} = \frac{2}{9} f^{\alpha}$ and $\left(g_j + g_k\right)^{SA} = \frac{1}{9} f^{\alpha}$, where the SA superscript indicates equilibrium values for the "standard alliance" case. We note that the allied parties' input contributions cannot be determined uniquely because of perfect substitutability. The probabilities of victory for the alliance and the non-allied player are $p_i^{NA} = \frac{2}{3}$ and $p_{jk}^{NA} = \frac{1}{3}$, respectively; the corresponding equilibrium payoffs are $\pi_i^{NA} = \frac{4}{9}$ and $\left(\pi_j + \pi_k\right)^{NA} = \frac{2}{9}$.

Relative to the no-alliance baseline model, the equilibrium allocations, probabilities, and payoffs establish the existence of an alliance formation puzzle. Alliance formation fails to improve the equilibrium likelihood of victory for the allies and leaves them with no greater (combined) expected payoff, even assuming the two allies can somehow overcome the free-rider problem in choosing individual inputs. Supposing zero transaction costs to forming the alliance, it is unclear why both parties would agree to collaborate in the first place, and any positive transaction costs would make such an arrangement strictly less appealing than the unallied baseline. Further, alliance formation doubles the equilibrium likelihood of victory for the *unallied* party and quadruples its payoff relative to the unallied baseline case, making the formation of the alliance all the more puzzling.

3.4 Complementary alliance with exogenous prize division

We again consider a two-party alliance that divides the contest prize exogenously and evenly in the case of victory in a three-party contest. Here, however, the allies' costs are complementary as follows.

$$\max_{g_i} \pi_i = \frac{g_i}{g_i+g_j+g_k} \cdot 1 - \frac{1}{f_i^{\alpha}} g_i \tag{7}$$

$$\max_{g_j} \pi_j = \frac{g_j+g_k}{g_i+g_j+g_k} \cdot \frac{1}{2} - \frac{1}{\left(f_j+f_k\right)^{\alpha}} g_j \tag{8}$$

$$\max_{g_k} \pi_k = \frac{g_j+g_k}{g_i+g_j+g_k} \cdot \frac{1}{2} - \frac{1}{\left(f_j+f_k\right)^{\alpha}} g_k \tag{9}$$

Since the contest success function remains the same for both the allies and the unallied party, the unallied party's objective function (7) is the same as in (4). The allies' objectives in (8) and (9), however, differ from (5) and (6) in their cost components, as now the alliance enhances cost efficiency.

Again letting $f_i = f_j = f_k = f$, we solve for the equilibrium allocations from the corresponding first-order conditions. In the case at hand, we find equilibrium input allocations of $g_i^{CE} = \frac{2^{1-\alpha} f^{\alpha}}{(2^{1-\alpha}+1)^2}$ and $\left(g_j + g_k\right)^{CE} = \frac{f^{\alpha}}{(2^{1-\alpha}+1)^2}$, probabilities of victory $p_i^{CE} = \frac{2^{1-\alpha}}{(2^{1-\alpha}+1)}$ and $p_{jk}^{CE} = \frac{1}{(2^{1-\alpha}+1)}$, and equilibrium payoffs $\pi_i^{CE} = \frac{2^{2-2\alpha}}{(2^{1-\alpha}+1)^2}$, and

$\left(\pi_j + \pi_k\right)^{CE} = \frac{2^{\alpha}+1}{2^{\alpha}(2^{1-\alpha}+1)^2}$, where the superscripts CE indicate the "complementary-exogenous" case. While the allies' individual input choices remain indeterminate, it is worthwhile to compare this case to the previous one to identify the effect of cost complementarity on its own.

When $\alpha = 0$, no complementarity between the two allies' inputs is assumed. As such, it can be verified that the results of subsection 3.2 hold. For that value of α, the sum of allied payoffs is no greater than in the previous case, and the likelihood that the alliance wins the contest equals $\frac{1}{3}$. In other words, the alliance formation puzzle remains for $\alpha = 0$. If we attach even small transaction costs to alliance formation here, it remains unclear why alliances form. For any $\alpha \in (0, 1)$, however, the payoff element of the alliance formation puzzle vanishes. That is, any extent of cost complementarity raises the sum of allied payoffs, as compared to in the no-alliance case; for $\alpha \in (0, 1)$, it can be verified that $\left(\pi_j + \pi_k\right)^{CE} = \frac{2^{\alpha}+1}{2^{\alpha}(2^{1-\alpha}+1)^2}$ exceeds the baseline payoff of $\frac{2}{9}$. The reason is not because the collective action problem goes away, however. In addition to input choices still not being uniquely determined for the individual allies, the sum of allied contest input spending is less than half that of the no-alliance case, which explains why the unallied party ends up better off than in the no-alliance scenario. Furthermore, the likelihood of allied victory does not exceed one-half for any α under consideration. Alliances overcome the formation puzzle only because the (cost) complementarity effect on payoffs dominates the collective action effect.

3.5 Complementary alliance with proportional prize division

Prize division is now determined by the following proportional division rule: *the prize is divided according to within-alliance input-contribution shares*. If, for example, an ally contributes one-third of the alliance's total inputs to the contest, it receives one-third of the prize in the event of allied victory. As with any sharing rule, including fixed division, proportional sharing might be adopted to preserve the alliance's future value.

If the alliance's benefits continue beyond the contest in question, the allies may share a general understanding that compliance with a proportional sharing rule can stabilize membership, a possibility we stress in the paper's conclusion. Alternatively, an outside party or superpower that aligns with both allies can enforce a proportional prize-sharing agreement to maintain the strength of a broader treaty organization. A superpower mandate, such as the Monroe Doctrine, for example, can effectively create hegemonic coercion of regional treaty partners against external threats.

The objective functions in a three-party contest comprising two allies supplying complementary inputs and agreeing to divide the contest's prize proportionally are as follows.

$$\max_{g_i} \pi_i = \frac{g_i}{g_i+g_j+g_k} \cdot 1 - \frac{1}{f_i^{\alpha}} g_i \tag{10}$$

$$\max_{g_j} \pi_j = \frac{g_j+g_k}{g_i+g_j+g_k} \cdot \frac{g_j}{g_j+g_k} - \frac{1}{(f_j+f_k)^{\alpha}} g_j \tag{11}$$

$$\max_{g_k} \pi_k = \frac{g_j+g_k}{g_i+g_j+g_k} \cdot \frac{g_k}{g_j+g_k} - \frac{1}{(f_j+f_k)^{\alpha}} g_k \tag{12}$$

The unallied party's (*i*) objective remains the same as in the previous cases, but for the allied parties (*j* and *k*), equations (11) and (12) differ from (8) and (9) because of the change in the prize-sharing rule. Rather than exogenously imposed as an equal split, each of the allies' prize shares, in the event of victory, now is determined by its input contribution.

We again let $f_i = f_j = f_k = f$ and solve for the following equilibrium allocations from first-order conditions. Unlike an exogenously adopted equal contest prize division, proportional division allows the individual ally's input contributions to be determined uniquely rather than as before only at the alliance level. When input contributions govern prize shares, we are able to solve a set of three first-order conditions in three unknowns. Doing so, we find equilibrium inputs of $g_i^{CP} = (2^{1-\alpha} - 1) \cdot \frac{2f^\alpha}{(2^{1-\alpha}+1)^2}$ and $g_j^{CP} = g_k^{CP} = \frac{2f^\alpha}{(2^{1-\alpha}+1)^2}$, meaning that $\left(g_j + g_k\right)^{CP} = \frac{4f^\alpha}{(2^{1-\alpha}+1)^2}$, where the CP superscripts indicate the "complementary-proportional" case. The equilibrium probabilities of victory are $p_i^{CP} = \frac{(2^{1-\alpha}-1)}{(2^{1-\alpha}+1)}$ and $p_{jk}^{CP} = \frac{2}{(2^{1-\alpha}+1)}$, implying that $p_{jk}^{CP} > \frac{2}{3}$ for all $\alpha \in (0,1)$. Accordingly, the equilibrium payoffs are $\pi_i^{CP} = \frac{\left(2^{1-\alpha}-1\right)^2}{(2^\alpha+1)^2}$ and $\pi_j^{CP} = \pi_k^{CP} = \frac{2^{1-\alpha}}{(2^\alpha+1)^2} > \frac{2}{9}\ \forall \alpha \in (0,1)$. The sum of the allies' benefits is $\left(\pi_j + \pi_k\right)^{CP} = \frac{2\cdot(2^{1-\alpha})}{(2^\alpha+1)^2} > \frac{4}{9}\ \forall \alpha \in (0,1)$. That is, the allied parties are better off individually and collectively than in any of the previous cases, including conflict with no alliances.

In the symmetric case we consider, allied parties choose the same input allocations, meaning that they end up sharing the prize equally (if they win), just as in the exogenous, equal division case. However, the collective action problem vanishes and allied parties, j and k, benefit from cost complementarity for all $\alpha \in (0,1)$. In this case, however, allies *choose* equal division endogenously through the proportional division rule. In the exogenous, equal division case, equal division is specified outside of the strategic interaction (i.e., by constitution) such that the free-ridership problem emerges. Knowing that equal division is assured, allies free ride on the efforts of one another. In the present case of proportional prize sharing, the division proportions are decided endogenously. Equal division, even in the symmetric case, is not specified but, rather, made possible through the agreement. Allies are able to arrive at the same equal division rule while also eradicating the collective action problem and raising expected payoffs in this treatment. Hence, an alliance with cost complementarity and an enforceable proportional division rule fully overcomes the alliance formation puzzle.

3.6 Summary comparisons

Table 1: Comparisons of equilibrium values

	Value	Ranking
Unallied party	Spending	$g_i^{CP} \leq g_i^{NA} = g_i^{SA} \leq g_i^{CE}$
	Probability of victory	$p_i^{CP} \leq p_i^{NA} < p_i^{CE} \leq p_i^{SA}$
	Payoffs	$\pi_i^{CP} \leq \pi_i^{NA} < \pi_i^{CE} \leq \pi_i^{SA}$
Allied parties	Spending	$(g_j + g_k)^{SA} \leq (g_j + g_k)^{CE} < (g_j + g_k)^{NA} \leq (g_j + g_k)^{CP}$
	Probability of victory	$p_{jk}^{SA} \leq p_{jk}^{CE} < (p_j + p_k)^{NA} \leq p_{jk}^{CP}$
	Payoffs	$(\pi_j + \pi_k)^{SA} = (\pi_j + \pi_k)^{NA} \leq (\pi_j + \pi_k)^{CE} < (\pi_j + \pi_k)^{CP}$
Total	Spending	$\sum_{z\in\{i,j,k\}} g_z^{SA} \leq \sum_{z\in\{i,j,k\}} g_z^{CE} < \sum_{z\in\{i,j,k\}} g_z^{NA} \leq \sum_{z\in\{i,j,k\}} g_z^{CP}$
	Payoffs	$\sum_{z\in\{i,j,k\}} \pi_z^{NA} < \sum_{z\in\{i,j,k\}} \pi_z^{CP} < \sum_{z\in\{i,j,k\}} \pi_z^{CE} < \sum_{z\in\{i,j,k\}} \pi_z^{SA}$

Notes: The superscripts denote the no-alliance baseline (NA), a standard alliance with (exogenously determined) equal prize sharing (SA), a complementary input alliance with exogenous equal prize sharing (CE), and a complementary input alliance with endogenously determined proportional prize sharing (CP).

To summarize our results and put them in context, we compare equilibrium outcomes across the four models in Table 1. From the second-to-last row, total conflict spending is lowest in the SA case followed by the CE case, because the free-rider effect on the allied parties' input choices (see fourth row) is strong enough to reduce total conflict spending. The CP case results in the most input spending because the allies are both motivated by the proportional division rule and advantaged by their cost complementarities, and the unallied party must respond accordingly. In that case, the allies' combined spending approaches its maximum of f^{α} as the cost-complementarity parameter α approaches 1, while the unallied party's input choice approaches zero.[7] The allies' probabilities of victory are commensurate with their input spending, with the SA model giving them the smallest chance and CP the largest chance of winning (fourth and fifth rows); the reverse ranking applies for the unallied party's p_i (second row of Table 1).

In terms of payoffs, the sixth row of Table 1 indicates that the allies are collectively worst off in the SA case and best off in the CP case, with the reverse being true for the unallied party (third row). The unallied party is best off when the allies free ride the most, while the allies are best off when their collective action problem is solved. Somewhat surprisingly, the CE case is second-best for both the allies and the unallied party: the allies are better off than in the NA case owing to cost complementarities, but since the fixed division rule still results in free riding, the unallied party's payoff ends up larger than NA as well. Finally, total payoffs (economic rents) are lowest in the NA case and highest in the SA case, as listed in the last row of Table 1. The formation of any alliance raises the sum of payoffs to the conflicting parties relative to NA, but of the alliance models, CP returns the smallest total payoff because it also entails the most total spending.

In no case do the three parties do as well as they could with an equal settlement without any conflict if such an outcome were possible in the absence of transaction costs. In fact, the only time any party receives an expected payoff exceeding $\frac{1}{3}$ of the spoils of victory individually is in the SA model, in which case the unallied party

[7] In the CP case, the allies' spending alone approaches full dissipation of the prize's value as the cost-complementarity parameter α approaches 1, while the unallied party's input choice approaches zero. At $\alpha = 1$, the game's equilibrium in this case is in mixed strategies, with the unallied party placing positive mass at zero spending.

benefits from the collective action problem suffered by the allies. Thus, while the CE and CP models can provide solutions to the alliance-formation puzzle, war's inefficiency remains. We can explain why alliances may form given that a conflict will take place, but not why conflict breaks out in the first place.

4 Conclusion

It is well-established that standard Tullock contest alliances (e.g., in conflict or rent-seeking settings) elicit free-riding. In a three-party contest where two of the parties ally, free riding is sufficiently strong to lower the alliance's likelihood of victory, undermine expected payoff gains within the alliance, and generate net benefits for the *unallied* party. Under the standard treatment, a collective action problem arises because allies treat one another's contest inputs as substitutes and exogenously divide the contest's prize equally. As an alternative, we consider a two-party alliance in a three-party contest that features both input cost complementarity and a proportional prize-sharing agreement that endogenously determines allies' shares of the prize in the event of victory based on their individual input choices. Input cost complementarity causes the alliance formation puzzle to disappear conditionally by allowing the allies to exploit input-cost savings but does not fully correct the collective action problem. The addition of proportional prize sharing, however—whether by formal means, such as a treaty or constitution, or by less formal but mutual understanding of fairness considerations, as we discuss subsequently—fully corrects the collective action problem. Taken together, the two measures unconditionally generate expected gains for the allied parties.

Comparing our model's results to the historical experience of NATO, discussed in Section 2, it makes sense that the alliance had less free-riding and more cooperation when the benefits of alliance membership fit more closely with their individual spending decisions. One of the main thrusts of the humanomics literature, stemming from the model of human behavior put forth by Adam Smith himself in his *Theory of Moral Sentiments* (1759), is that our behavior is guided by a tacit understanding of our environment and its *general rules of conduct* (Smith & Wilson, 2017; 2018). Our actions are governed by our own, internal "impartial spectator" (Smith & Wilson, 2014), or personal narrative (Osborn, Wilson, & Sherwood, 2015). Combined with our results on the effects of proportional prize-sharing agreements, these ideas help explain why free-riding among NATO members was not prevalent prior to the early 2000s, but has once again become an issue, even after an explicit pledge by members to commit 2% of their GDPs to defense spending (Kim & Sandler, 2020).[8]

During the flexible response era, running from 1967 to 1990, a shared understanding emerged among NATO allies that failing to deploy sufficient conventional forces could make them the alliance's weakest link that would draw an attack (Sandler & Murdoch, 2001; Sandler & Hartley, 2001). Allies implicitly knew that the benefits of the alliance depended at least in part on their own spending contributions. Since then, as weapons have become more technologically advanced, defense efforts aimed at more deterrent-themed efforts (against terrorist threats from the Middle East, for example), the benefits of alliance membership decoupled from individual spending efforts, and free riding resumed as in the 1949-1966 era when nuclear deterrence was the chief concern. Despite the 2% spending pledge made in 2014, in part a response to Russia's (at the time) recent invasion of Crimea, many NATO allies continued to reduce their military spending even as Russia increased its own (George & Sandler, 2022). The common understanding seems to have reverted to the idea that one or a few of the larger members can (and will) spend sufficiently on deterrence measures so that all others can enjoy the benefits without spending themselves.

Whether more recent events in Ukraine have changed NATO members' shared understanding of how their own defense spending contributes to the benefits of alliance membership remains to be seen. We have shown here that proportional prize-sharing agreements or arrangements can help deal with the free riding that leads to the alliance

[8]See https://www.nato.int/cps/en/natohq/topics_49198.htm (last accessed July 31st, 2024) for more on the pledge itself.

puzzle. Whether the agreement is codified explicitly, as in a treaty or constitution, or based on a shared understanding of conduct, a connection between the benefits of alliance membership and individual members' contributions appears to be crucial for the overall success of the alliance. If alliance members believe they will receive the same benefit regardless of individual spending, they behave accordingly by free riding. Thus, rather than a fixed target (or pledge) for spending contributions irrespective of benefits, a future NATO—or any alliance—would be better advised to incentivize its members with an understanding of how those benefits relate to spending.

References

Arino, A., & Ring, P. S. (2010). The role of fairness in alliance formation. *Strategic Management Journal*, 31(10), 1054-1087.

Blattman, C. (2022). *Why we fight: The roots of war and the paths to peace*. New York, NY: Viking.

Boudreau, J. W., Rentschler, L., & Sanders, S. (2019). Stag hunt contests and alliance formation. *Public Choice*, 179, 267-285.

Boudreau, J. W., Sanders, S., & Shunda, N. (2019). The role of noise in alliance formation and collusion in conflicts. *Public Choice*, 179, 249-266.

Chang, Y. M., Potter, J., & Sanders, S. (2007). War and peace: third-party intervention in conflict. *European Journal of Political Economy*, 23(4), 954-974.

Ehrlich, J., Harmon, M., & Sanders, S. (2020). The alliance formation puzzle in contests with capacity-constraints: A test using American football reception-coverage contest data. *Plos One*, 15(3), e0227750.

Elmuti, D., Abebe, M., & Nicolosi, M. (2005). An overview of strategic alliances between universities and corporations. *Journal of Workplace Learning*, 17(1/2), 115-129.

Fearon, J. D. (1995). Rationalist explanations for war. *International Organization*, 49, 379-379.

George, J., & Sandler, T. (2022). NATO defense demand, free riding, and the Russo-Ukrainian war in 2022. *Journal of Industrial and Business Economics*, 49, 783-806.

Kang, N., &. Sakai, K. (2000). International strategic alliances: Their role in industrial globalisation. *OECD Science, Technology and Industry Working Papers*, No. 2000/05, OECD Publishing, Paris,

Ke, C., Konrad, K. A., & Morath, F. (2013). Brothers in arms – An experiment on the alliance puzzle. *Games and Economic Behavior*, 77(1), 61-76.

Khanna, J., & Sandler, T. (1996). NATO burden sharing: 1960-1992. *Defence and Peace Economics*, 7(2), 115-133.

Kim, W., & Sandler, T. (2020). NATO at 70: Pledges, free riding, and benefit-burden concordance. *Defence and Peace Economics*, 31(4), 400-413.

Kim, W., & Sandler, T. (2024). NATO security burden sharing, 1991-2020. *Defence and Peace Economics*, 35(3), 265-280.

Konishi, H., & Pan, C. Y. (2020). Sequential formation of alliances in survival contests. *International Journal of Economic Theory*, 16(1), 95-105.

Konrad, K. A. (2012). Information alliances in contests with budget limits. *Public Choice*, 151, 679-693.

Konrad, K. A., & Kovenock, D. (2009). The alliance formation puzzle and capacity constraints. *Economics Letters*, 103(2), 84-86.

Leeson, P. T. (2011). Government, clubs, and constitutions. *Journal of Economic Behavior & Organization*, 80(2), 301-308.

Leeson, P. T., & Skarbek, D. B. (2010). Criminal constitutions. *Global Crime*, 11(3), 279-297.

Mathews, T., & Sanders, S. (2019). Strategic and experimental analyses of conflict and terrorism. *Public Choice*, 179(3-4), 169-174.

Mitchell, M. D. (2019). Rent seeking at 52: an introduction to a special issue of public choice. *Public Choice*, 181(1-2), 1-4.

Munger, M. C. (2019). Tullock and the welfare costs of corruption: there is a "political Coase Theorem." *Public Choice*, 181(1-2), 83-100.

Murdoch, J. C., & Sandler, T. (1984). Complementarity, free riding and the military expenditure of NATO allies. *Journal of Public Economics*, 25(1-2), 83-101.

Olson, M., & Zeckhauser, R. (1966). An economic theory of alliances. *Review of Economics and Statistics*, 48(3), 266-279.

Oneal, J. A., & Elrod, M. A. (1989). NATO burden sharing and the forces of change. *International Studies Quarterly*, 33(4), 435-456.

Osborn, J., Wilson, B. J., & Sherwood, B. R. (2015). Conduct in narrativized trust games. *Southern Economic Journal*, 81(3), 562–597.

Oyewole, S. (2015). Boko Haram: Insurgency and the war against terrorism in the Lake Chad region. *Strategic Analysis*, 39(4), 428-432.

Pirnie, B. R., Vick, A. J., Grissom, A. R., Mueller, K. P., & Orletsky, D. T. (2005). *Beyond close air support: Forging a new air-ground partnership*. Santa Monica, CA: RAND Corporation.

Sandler, T (1999). Alliance formation, alliance expansion, and the core. *Journal of Conflict Resolution*, 43(6), 727-747.

Sandler, T., & Forbes, J. F. (1980). Burden sharing, strategy, and the design of NATO. *Economic Inquiry*, 18(3), 425-444.

Sandler, T., & Hartley, K. (2001). Economics of alliances: The lessons for collective action. *Journal of Economic Literature*, 39(3), 869-896.

Sandler, T., & Murdoch, J. C. (2000). On sharing NATO defence burdens in the 1990s and beyond. *Fiscal Studies*, 21(3), 297-327.

Sandler, T., & Shimizu, H. (2014). NATO burden sharing 1999-2010. *Foreign Policy Analysis*, 10(1), 43-60.

Skaperdas, S. (1998). On the formation of alliances in conflict and contests. *Public Choice*, 96(1-2), 25-42.

Smith, A. (1759). *The theory of moral sentiments.* Liberty Fund: Indianapolis, IN (1982).

Smith, V. L., & Wilson, B. J. (2017). Sentiments, conduct, and trust in the laboratory. *Social Philosophy and Policy*, 34(1), 25-55.

Smith, V. L.. & Wilson, B. J. (2018). Equilibrium play in voluntary ultimatum games: Beneficence cannot be extorted. *Games and Economic Behavior*, 109, 452-464.

Smith, V. L., & Wilson, B. J. (2019). *Humanomics: Moral sentiments and the wealth of nations for the twenty-first century*. Cambridge, UK: Cambridge University Press.

Tullock, G. (1980) Efficient rent seeking. In Buchanan, J., Tollison, R., & Tullock, G. (eds), *Toward a theory of the rent-seeking society*, pp. 97–112. College Station: Texas A&M University Press.