KENNESAW STATE UNIVERSITY
COLLEGE OF SCIENCE AND MATHEMATICS
Department of Mathematics

THE 2022–2023 KENNESAW STATE UNIVERSITY HIGH SCHOOL MATHEMATICS COMPETITION

PART I – MULTIPLE CHOICE

For each of the following 25 questions, carefully blacken the appropriate box on the answer sheet with a #2 pencil. Do not fold, bend, or write stray marks on either side of the answer sheet. Each correct answer is worth 6 points. Two points are given if no box is marked. Zero points are given for an incorrect answer or if multiple boxes are marked. Note that wild guessing is likely to lower your score. When the exam is over, give your answer sheet to your proctor. You may keep your copy of the questions.

NO CALCULATORS

1. A 100-page newspaper consists of 25 folded sheets of paper, so each sheet has four pages. For example, the opening sheet has pages 1, 2, 99, and 100. If a sheet has page 16, what is the sum of the numbers on the other three pages of the sheet?

(A) 182 (B) 184 (C) 186 (D) 188 (E) None of these

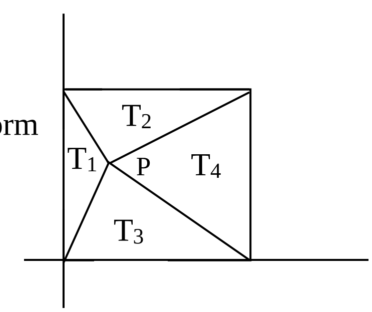
2. Given that the 9-digit number $k3kkkkkkk$ is a prime number for <u>at least</u> one digit k, for how many digits k will the number $k3kkkkkkk$ be prime?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. The product of the roots of $ax^2 + bx + c = 0$ is 6. The product of the roots of $bx^2 + cx + a = 0$ is 8. Compute the product of the roots of the equation $cx^2 + ax + b = 0$.

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) $\frac{4}{3}$ (D) $\frac{1}{48}$ (E) 48

4. A square of side length 10 units is positioned with one vertex at the origin and the opposite vertex at the point (10, 10). A point P is placed inside the square. Segments are drawn from point P to each vertex of the square to form four triangles T_1, T_2, T_3, T_4 as indicated in the diagram. If the ratio of the areas of $T_1 : T_2 : T_3 : T_4$ is 1 : 2 : 3 : 4, what are the coordinates of point P?



(A) (1.5, 5.5) (B) (1.5, 6) (C) (1, 6) (D) (2, 6) (E) (2.5, 6)

5. The word *ROBOT* represents a 5-digit number, with different letters representing different digits, and the same letters representing the same digit. Every digit is a prime number and so is the sum of the 5 digits. The 2-digit number *OR* and the 3-digit number *OBT* are also primes. What digit does the letter *B* represent?

(A) 2 (B) 3 (C) 5 (D) 7 (E) Cannot be determined

6. A farmer bought some chicks and paid a total of \$420. He paid the same amount for each chick. If each chick had cost a dollar more, he would have obtained 2 fewer chicks for the same amount of money. How many chicks did he buy?

(A) 20 (B) 24 (C) 28 (D) 30 (E) 32

7. What is the 2022nd digit (after the decimal point) in the decimal representation of $\frac{1}{7}$?

(A) 1 (B) 2 (C) 4 (D) 5 (E) 7

8. Dr. Garner organized the state high school math tournament. He noted that a total of 215 students attended the math tournament, of which 65 were freshmen, 77 were sophomores, 96 were juniors, and 82 were seniors. What base was Dr Garner using?

(A) 12 (B) 13 (C) 14 (D) 15 (E) None of these

9. The number N = 111...1 consists of 2022 ones. It is exactly divisible by 11. How many zeroes are there in the quotient $\frac{N}{11}$?

(A) 1010 (B) 1011 (C) 1012 (D) 1110 (E) 1111

10. Equilateral triangle ABC, with side length 7, is inscribed in a circle. The length of the chord $\overline{\mathrm{BD}}$ (chord: line segment joining any two points on the circle) of the circle is 8. Compute the <u>product</u> of all possible lengths of chord $\overline{\mathrm{AD}}$.

(A) 10 (B) 12 (C) 14 (D) 15 (E) 18

11. The polynomial $P(x)$ has minimal degree, real coefficients, and two of its zeros are $-2i$ and $3+i$. If $P(-1)=17$, compute $P(1)$.

(A) –17 (B) –10 (C) 5 (D) 10 (E) 15

12. Given that $(a+b)+(b+c)+(c+a)=18$, and $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{5}{9}$, compute the value of $\frac{c}{a+b}+\frac{a}{b+c}+\frac{b}{c+a}$.

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

13. Only one ordered pair (x, y) of real numbers satisfies the system

$$\sqrt{x-y}=x+y-7$$
$$\sqrt{x+y}=x-y-1.$$

Compute the product xy.

(A) 16 (B) 16.25 (C) 16.5 (D) 16.75 (E) None of these

14. Debbie and Don were comparing their stacks of pennies. Debbie said "If you gave me a certain number of pennies from your stack, then I'd have six times as many as you, but if I gave you that number, you'd have one-third as many as me." What is the smallest number of pennies that Debbie could have in her stack?

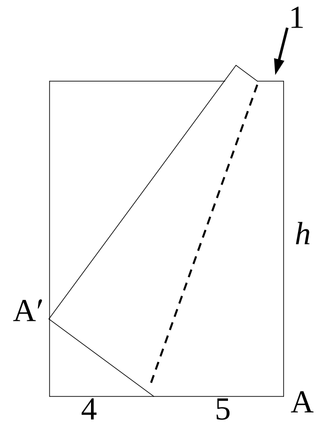
(A) 41 (B) 43 (C) 45 (D) 47 (E) 49

15. Let $S = \{1,3,5\ldots,63\}$. How many subsets of S have elements whose sum is 1000?

(A) 9 (B) 10 (C) 11 (D) 12 (E) None of these

16. The diagram at the right shows an *h* x 9 rectangular piece of paper. The paper is folded along the dotted line segment so that vertex A lies on the opposite side of the rectangle (at A′). Given the measurements shown, compute *h*, the length of the paper.

(A) 10 (B) 10.5 (C) 11 (D) 11.5 (E) 12



17. Suppose that $3^a = 4$, $4^b = 5$, $5^c = 6$, $6^d = 7$, $7^e = 8$, and $8^f = 9$. What is the value of the product $abcdef$?

(A) 1.2 (B) 2 (C) 2.5 (D) 3.6 (E) 4

18. Given two sequences, one arithmetic and one geometric, each containing three numbers. When the corresponding terms of each sequence are added, the sums are 60, 56, and 64, respectively. The sum of the three numbers of the arithmetic sequence is 96. There is more than one arithmetic sequence that satisfies these conditions. Compute the sum when the largest term of all such arithmetic sequences are added.

(A) 96 (B) 100 (C) 108 (D) 144 (E) None of these

19. Compute the number of three-digit positive integers *n* such that if *n* is added to the number formed by writing the three digits of *n* in reverse order, the sum is 1372.

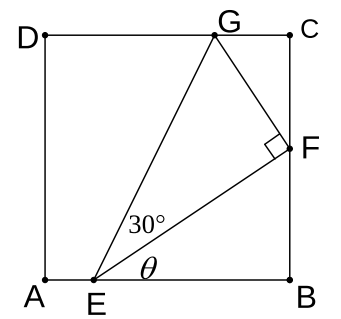
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

20. Players Abe, Bea and Cal take turns rolling a standard die. Abe starts and will win if he rolls a one. If he doesn't win, then Bea rolls and wins if she rolls a two or a three. If she doesn't win either, then Cal rolls and wins with a four, five or six. If no one wins then Abe tries again, etc. and the players alternate in the order Abe, Bea, Cal, Abe, Bea, Cal, Abe, ... until someone wins. What is the probability that Cal will win the game?

(A) $\frac{1}{3}$ (B) $\frac{5}{13}$ (C) $\frac{1}{2}$ (D) $\frac{3}{5}$ (E) None of these

21. Define a set of positive integers to be *balanced* if the set is not empty and the number of even integers in the set is equal to the number of odd integers in the set. How many subsets of the set of the first 10 positive integers are balanced?

(A) 251 (B) 225 (C) 146 (D) 31 (E) None of these



22. Given square ABCD with side length 1. The measure of $\angle GEF = 30°$, $\angle GFE$ is a right angle, and $\tan\theta = \frac{2}{3}$. Compute the length of $\overline{BF}$.

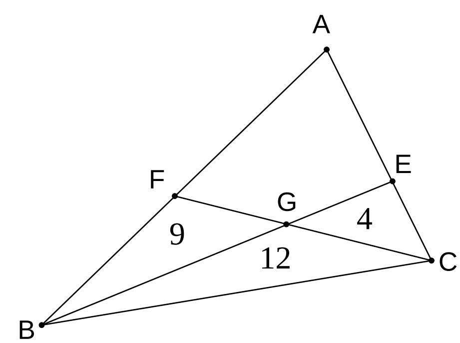
(A) $\frac{\sqrt{3}}{3}$ (B) $4 - 2\sqrt{3}$ (C) $\sqrt{3} - 1$ (D) $\frac{3-\sqrt{3}}{2}$ (E) None of these

23. A sequence $\{a_n\}$ is defined as follows: a_1 and a_2 are positive numbers and $a_n = \frac{1 + a_{n-1}}{a_{n-2}}$. If $a_1 = 20$ and $a_2 = 22$, compute the value of a_{2022}.

(A) 20 (B) 22 (C) $\frac{21}{22}$ (D) $\frac{23}{20}$ (E) None of these

24. Given that $x^2 - 3x + 1 = 0$, compute $x^9 + x^7 + x^{-9} + x^{-7}$.

(A) 27 (B) 213 (C) 2401 (D) 4959 (E) 6621



25. In the adjoining figure, points E and F are chosen on sides $\overline{AC}$ and $\overline{AB}$ of ΔABC, respectively, and G is the intersection of segments $\overline{BE}$ and $\overline{CF}$. The area of ΔBGF is 9, the area of ΔBGC is 12, and the area of ΔCGE is 4. Compute the area of quadrilateral AFGE.

(A) $\frac{37}{3}$ (B) $\frac{38}{3}$ (C) 13 (D) $\frac{40}{3}$ (E) 14

Solutions

1. C Pages on the same side of a sheet add to 101. Therefore, the four pages are 15, 16, 85 and 86. The desired sum is 186.

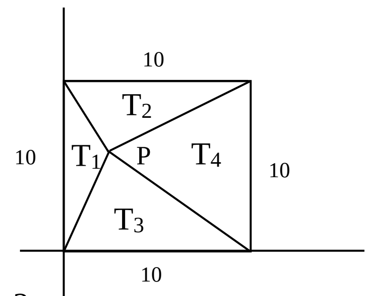
2. A Any even value of *k* will make an even number, and 535555555 is divisible by 5. Both 3 and 9 make the number divisible by 3, and the divisibility test for 11 shows that 11 divides it when *k* equals 7. The only value of *k* left is 1, and since we are told that the number is prime for at least one value of *k*, *k* must equal 1.

3. D The product of the roots of $ax^2+bx+c=0$ is $\frac{c}{a}=6 \Rightarrow c=6a$

The product of the roots of $bx^2+cx+a=0$ is $\frac{a}{b}=8 \Rightarrow b=\frac{a}{8}$

The product of the roots of $cx^2+ax+b=0$ is $\frac{b}{c}$, and $\frac{b}{c}=\frac{\frac{a}{8}}{6a}=\frac{1}{48}$.

4. D Represent the four areas by a, $2a$, $3a$ and $4a$. Since the area of the square is 100, $a+2a+3a+4a=100$ and $a=10$.



Using the formula for the area of a triangle $A=\frac{1}{2}bh$ on triangle T_3 whose area is 30, we obtain $h=6$, the distance from P to the *x*-axis. Using triangle T_1 whose area is 10, we obtain $h=2$, the distance from P to the *y*-axis. Therefore, the coordinates of point P are (2, 6).

5. C The four different letters must stand for the four one-digit primes 2, 3, 5, 7 in some order. Since the sum of these four numbers is 17, the only way the sum of the five numbers can be odd (necessary to make it prime) is if the duplicated number is 2. Therefore *O* stands for 2. Now *R* cannot be 5 or 7 (*OR* would not be prime), so *R* must be 3. Also, *T* cannot be 5 (*OBT* would not be prime), so $T=7$, leaving $B=5$. Checking, $3+2+5+2+7=19$, and 23 and 257 all are primes, so *B* represents 5.

6. D Let n = number of chicks, c = cost of each chick. Then $nc=420$ and $(n-2)(c+1)=420$.

Therefore, $nc=nc-2c+n-2$ or $c=\frac{n-2}{2}$. Substituting,

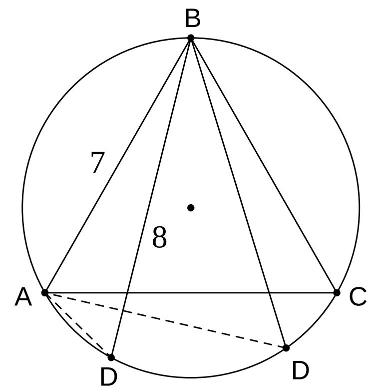
$nc=\frac{n^2-2n}{2}=420 \Rightarrow n^2-2n-840=0 \Rightarrow (n-30)(n+28)=0$, and $n=30$.

7. E $\frac{1}{7}=.\overline{142857}$ has a six-digit repeat. Since $2022=6(337)$, the 2022nd digit of the decimal representation of $\frac{1}{7}$ is the same as the last digit of the six-digit repeat, which is 7.

8. D If Dr. Garner is using base b, then $65_b+77_b+96_b+82_b=215_b \Rightarrow$
$6b+5+7b+7+9b+6+8b+2=2b^2+b+5$. Therefore, $2b^2-29b-15=0$.
Factoring, $(2b+1)(b-15)=0$, and $b=15$.

9. A Let's look for a pattern: $\frac{1111}{11} = 101$, $\frac{111111}{11} = 10101$, $\frac{11111111}{11} = 1010101$. When the number N has n- ones, where n is even the number of ones in $\frac{N}{11}$ is one-half n, and the number of zeros is one less than this. Therefore, if N has 2022 ones, then $\frac{N}{11}$ has 1010 zeros.

10. D There are two possible locations for point D on minor arc AC, as shown. In either case, the measure of $\angle$BDA is 60°. Letting AD = x, and using the Law of Cosines on ΔBAD, and noting $\cos 60 = \frac{1}{2}$,



$$7^2 = 8^2 + (x)^2 - 2(8)(x)\cos 60$$
$$49 = 64 + x^2 - 8x \quad \Rightarrow \quad x^2 - 8x + 15 = 0$$

Therefore, x = 3 or 5, and the desired product is 15.

11. C Since the coefficients of $P(x)$ are real, $2i$ and $3 - i$ are also zeros and $P(x)$ has degree 4. Then $P(x) = a(x - 2i)(x + 2i)[x - (3 + i)](x - [3 - i]) = a(x^2 + 4)(x^2 - 6x + 10)$. Since $P(-1) = a(5)(17) = 17$, $a = \frac{1}{5}$, and $P(1) = \frac{1}{5}(5)(5) = 5$.

12. A We are given $(a + b) + (b + c) + (c + a) = 18$, and so $a + b + c = 9$. Multiplying this last equation by the other given equation, $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = \frac{5}{9}$, we get $\frac{a+b+c}{a+b} + \frac{a+b+c}{b+c} + \frac{a+b+c}{c+a} = 5$. Therefore,

$$\frac{c}{a+b} + \frac{a}{b+c} + \frac{b}{c+a} = \left(\frac{a+b+c}{a+b} - 1\right) + \left(\frac{a+b+c}{b+c} - 1\right) + \left(\frac{a+b+c}{c+a} - 1\right) = 5 - 3 = 2.$$

13. B Let $a = \sqrt{x - y}$ and $b = \sqrt{x + y}$. Then the given equations become $a = b^2 - 7$ and $b = a^2 - 1$. Substituting, $a = (a^2 - 1)^2 - 7 \Rightarrow a^4 - 2a^2 - a - 6 = 0$. Factoring, we obtain $(a - 2)(a^3 + 2a^2 + 2a + 3) = 0$. The only real solution with $a \geq 0$ is $a = 2$, from which $b = 3$. Therefore, $\sqrt{x - y} = 2 \Rightarrow x - y = 4$ and $\sqrt{x + y} = 3 \Rightarrow x + y = 9$. Finally, $x = \frac{13}{2}$, $y = \frac{5}{2}$, and the product is 16.25.

14. C Let a and b be, respectively, the number of pennies that Debbie and Don had, and let x be the certain number of pennies. From the given information, we obtain the following two equations:

$$a + x = 6(b - x) \quad \text{and} \quad a - x = 3(b + x).$$

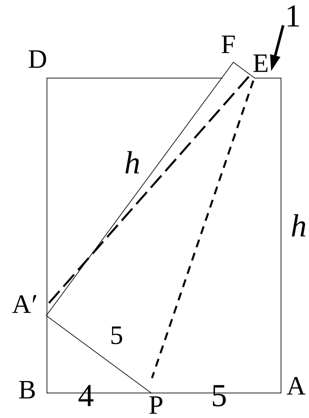
From the first equation, $a = 6b - 7x$, and from the second equation, $a = 3b + 4x$. Therefore,

$$6b - 7x = 3b + 4x \quad \Rightarrow \quad b = \frac{11}{3}x.$$

Since a, b, and x are required to be positive integers, the smallest possible value for x is 3. Since $a = 3b + 4x = 3 \cdot \frac{11}{3} + 4x = 15x$, the smallest number of pennies that Debbie could have had is $15 \cdot 3 = 45$.

15. C Note that since S contains the first 32 positive odd integers, the sum of its elements is $32^2 = 1024$. Thus, the problem is equivalent to finding the number of subsets of S whose elements each have a sum of 24. We can count these by hand: {23,1}, {21,3}, {19,5}, {17,7}, {15,9}, {15,5,3,1}, {13,11}, {13,7,3,1}, {11,7,5,1}, {11,9,3,1}, and {9,7,5,3}. Therefore, there are 11 such subsets.

16. E Label the diagram as shown and note that EF = 1, ED = 8, PA' = 5, and FA′= h. Construct $\overline{\text{EA}'}$. Because ΔPBA′ is a right triangle, BA′ = 3, making DA′ = h – 3.
Using the Pythagorean Theorem on ΔDEA′, $(h-3)^2 + 8^2 = (\text{EA}')^2$
Using the Pythagorean Theorem on ΔFEA′, $h^2 + 1^2 = (\text{EA}')^2$.
Clearing parentheses, subtracting the two equations and solving for h gives h = 12.



17. B Converting each equation into logarithmic form and using the change of base formula:

$$abcdef = \left(\frac{\log 4}{\log 3}\right)\left(\frac{\log 5}{\log 4}\right)\left(\frac{\log 6}{\log 5}\right)\left(\frac{\log 7}{\log 6}\right)\left(\frac{\log 8}{\log 7}\right)\left(\frac{\log 9}{\log 8}\right) = \frac{\log 9}{\log 3} = 2.$$

18. B <u>Method 1</u>: Let the terms of the arithmetic sequence be $a - d$, a, $a + d$. Thus, $3a = 96$, and $a = 32$, making the terms of the arithmetic sequence $32 - d$, 32, $32 + d$. Let the terms of the geometric sequence be $\frac{b}{r}$, b, br. Therefore, $32 + b = 56$, and $b = 24$, making the terms of the geometric sequence $\frac{24}{r}$, 24, $24r$. Now $32 - d + \frac{24}{r} = 60$ and $32 + d + 24r = 64$. Adding these two equations gives $64 + \frac{24}{r} + 24r = 124$ or $2r^2 - 5r + 2 = 0$. Thus r $= \frac{1}{2}$ or 2. Thus, there are two possible geometric sequences: 48, 24, 12 (when $r = \frac{1}{2}$), and 12, 24, 48 (when $r = 2$). The corresponding arithmetic sequences are 12, 32, 52 and 48, 32, 16 and the desired sum is 52 + 48 = 100.

<u>Method 2</u>: The arithmetic progression is 32-d, 32, 32+d. Using the sums of the corresponding terms, we get the geometric progression being 28+d, 24, 32-d. Then, one simply has (28+d)(32-d) = 24^2 which gives the quadratic equation d^2 - 4d - 320 = 0. Solving by factoring, we have d = 20 or d = -16. These give two possible arithmetic progressions 12, 32, 52 or 48, 32, 16. Hence, the answer is 52+48=100.

19. E <u>Method 1</u>: We will use the notation abc for a three-digit number with digits a, b and c in this order. Let abc be a three-digit number such that $abc + cba = 1372$. Then

$$1372 = (100a + 10b + c) + (100c + 10b + a) = 101(a + c) + 20b.$$

This gives $101(a + c) = 1372 - 20b$. Since $1372 - 20b = 4(343 - 5b)$, the number $a + c$ must be divisible by 4. Because $1 \le a + c \le 18$, the possible choices for $a + c$ are 4, 8, 12, and 16. Of these choices, only $a + c = 12$ gives an integer for $b = \frac{1}{20}[1372 - 101(a + c)]$ which is $b = 8$. There are 7 pairs of digits (a, c) such that $a + c = 12$: (3, 9), (4, 8), (5, 7), (6, 6), (7, 5), (8, 4), and (9, 3). This gives 7 possible numbers for n: 389, 488, 587, 686, 785, 884, and 983. One can check that all of these satisfy the requirements.

Method 2: Let $\overline{abc}$ be a three-digit number such that:
$\overline{abc}$ +
$\overline{cba}$
1372
Then $a + c = 2$ or 12. Looking at the sum of the last two digits, $\rightarrow a + c = 12$
So, There are 7 pairs of digits (a, c): (3, 9), (4, 8), (5, 7), (6, 6), (7, 5), (8, 4), and (9, 3).
Also, because $2b + 1 = 17 \rightarrow b = 8$.

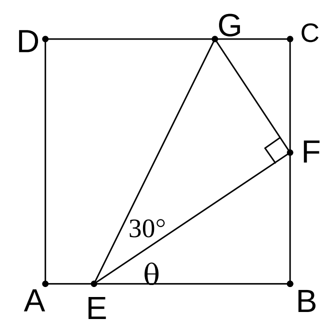
20. B Method 1: Since each roll of the die is independent of the others, the probability that Cal wins on his first roll is equal to the probability that Abe loses times the probability that Bea loses times the probability that player Cal wins, which is $\left(\frac{5}{6}\right)\cdot\left(\frac{2}{3}\right)\cdot\left(\frac{1}{2}\right) = \frac{5}{18}$. This is also the probability that nobody wins on the first round. Let p denote the probability that Cal wins the whole game. This can happen in two ways, either he wins on his first roll, or he wins later. This yields the following equation:

P(Cal wins in first round) + P(nobody wins in the first round)·P(Cal wins later) = P(Cal wins) or $\frac{5}{18} + \frac{5}{18}\cdot p = p$, and this equation gives $p = \frac{5}{13}$.

Method 2: . As in the previous solution we can calculate the probability that Cal wins in the first round to be $\frac{5}{18}$. The probability that Cal wins on his second roll is the probability that all three players lose on their first rolls, which is the same $\frac{5}{18}$, times the probability that player Cal then wins on his next roll, which is also $\frac{5}{18}$. This probability is thus $\left(\frac{5}{18}\right)^2$. Continuing in this manner, the total probability that player Cal wins is the sum of the infinite geometric series $\frac{5}{18} + (\frac{5}{18})^2 + (\frac{5}{18})^3 + \ldots = \frac{\frac{5}{18}}{1-\frac{5}{18}} = \frac{5}{13}$.

21. A The set of the first ten positive integers contains five odd integers and five even integers. Therefore, there are ${}_5C_k$ ways to choose k odd integers from five odd integers,and also there are ${}_5C_k$ ways to choose k even integers from five even integers. Therefore,there are $({}_5C_k)^2$ ways to pick a balanced subset containing k odd integers and k even integers.
Therefore, the answer is
$({}_5C_1)^2 + ({}_5C_2)^2 + ({}_5C_3)^2 + ({}_5C_4)^2 + ({}_5C_5)^2 = 5^2 + 10^2 + 10^2 + 5^2 + 1 = 251$.

22. B Since $\angle CFG$ is complementary to $\angle EFB$, and $\angle FEB$ is complementary to $\angle EFB$, $\angle CFG \cong \angle FEB$, making $\Delta EFB \sim \Delta FGC$ and $\angle GFC = \theta$. Noting that $\tan 30° = \frac{1}{\sqrt{3}}$, we have $\frac{FB}{GC} = \frac{EF}{GF} = \sqrt{3}$.
Letting $GC = x$, $FB = x\sqrt{3}$, and $CF = 1 - x\sqrt{3}$.
Then $\tan\theta = \tan(\angle GFC) = \frac{x}{1-x\sqrt{3}} = \frac{2}{3}$. Solving for x,



$2 - 2x\sqrt{3} = 3x \quad \Rightarrow \quad 2 = x(2\sqrt{3} + 3) \quad \Rightarrow \quad x = \frac{2}{2\sqrt{3}+3} = \frac{4\sqrt{3}-6}{3}$

Therefore, BF = $\left(\frac{4\sqrt{3}-6}{3}\right)\sqrt{3} = \frac{12-6\sqrt{3}}{3} = 4 - 2\sqrt{3}$.

23. B Let's examine the first few terms of the sequence. $a_n = \frac{1 + a_{n-1}}{a_{n-2}}$ when $a_1 = 20,$ and $a_2 = 22.$

$a_1 = 20,\ a_2 = 22,\ a_3 = \frac{1+22}{20} = \frac{23}{20},\ a_4 = \frac{1+\frac{23}{20}}{22} = \frac{43}{(20)(22)},$

$a_5 = \frac{1+\frac{43}{(20)(22)}}{\frac{23}{20}} = \frac{483}{(22)(23)} = \frac{21}{22},\ a_6 = \frac{1+\frac{21}{22}}{\frac{43}{(20)(22)}} = 20,\ a_7 = \frac{1+20}{\frac{21}{22}} = 22.$

Therefore, $a_1 = a_6$ and $a_2 = a_7$, and the sequence repeats every 5 terms.
Since $2022 \equiv 2 \bmod 5$, $a_{2022} = a_2 = 22$.

24. E Since $x \neq 0$, the given equation, $x^2 - 3x + 1 = 0$ can be rewritten as $x + \frac{1}{x} = 3$.

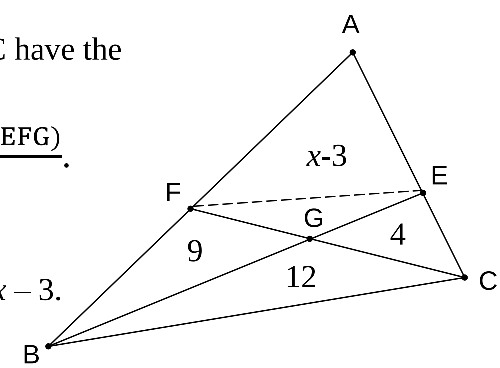
Now, $(x + \frac{1}{x})^2 = x^2 + \frac{1}{x^2} + 2 = 9 \Rightarrow x^2 + \frac{1}{x^2} = 7$.

Repeating this process twice more, $(x^2 + \frac{1}{x^2})^2 = x^4 + \frac{1}{x^4} + 2 = 49 \Rightarrow x^4 + \frac{1}{x^4} = 47$

and $(x^4 + \frac{1}{x^4})^2 = x^8 + \frac{1}{x^8} + 2 = 2209 \Rightarrow x^8 + \frac{1}{x^8} = 2207$.

Now $x^9 + x^7 + x^{-9} + x^{-7} = (x + \frac{1}{x})(x^8 + x^{-8}) = 3(2207) = 6621$.

25. A Construct segment $\overline{EF}$. Since ΔBFG and ΔBGC have the same altitude from B, $\frac{\text{area}(\Delta BFG)}{\text{area}(\Delta BGC)} = \frac{FG}{GC}$. Similarly, since ΔEFG and ΔEGC have the same altitude from E, $\frac{\text{area}(\Delta EFG)}{\text{area}(\Delta EGC)} = \frac{FG}{GC}$. Thus, $\frac{9}{12} = \frac{\text{area}(\Delta EFG)}{4}$.



Therefore, area(ΔEFG) = 3.

Let x be the area of quadrilateral AFGE. Then area(ΔAFE) = $x - 3$.

Since ΔAFE and ΔFBE have the same altitude from E,

$\frac{\text{area}(\Delta AFE)}{\text{area}(\Delta FBE)} = \frac{AF}{FB}$. Similarly, since ΔAFC and ΔFBC have the same altitude from C,

$\frac{\text{area}(\Delta AFC)}{\text{area}(\Delta FBC)} = \frac{AF}{FB}$. Therefore, $\frac{x+4}{21} = \frac{x-3}{12}$, from which $x = \frac{37}{3}$.